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ABSTRACT

This volume is the sixteenth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by second-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of five groups of lessons. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. This unit begins with a review of ordering concepts and then introduces linear measurement to the nearest half inch, measuring and adding fractional units, and measuring diameters and circumferences. Fourteen lessons are devoted to systems of numeration and place value. The measurement of weight, an introduction to negative numbers, and our monetary system are the subjects of other lesson sequences. (SD)

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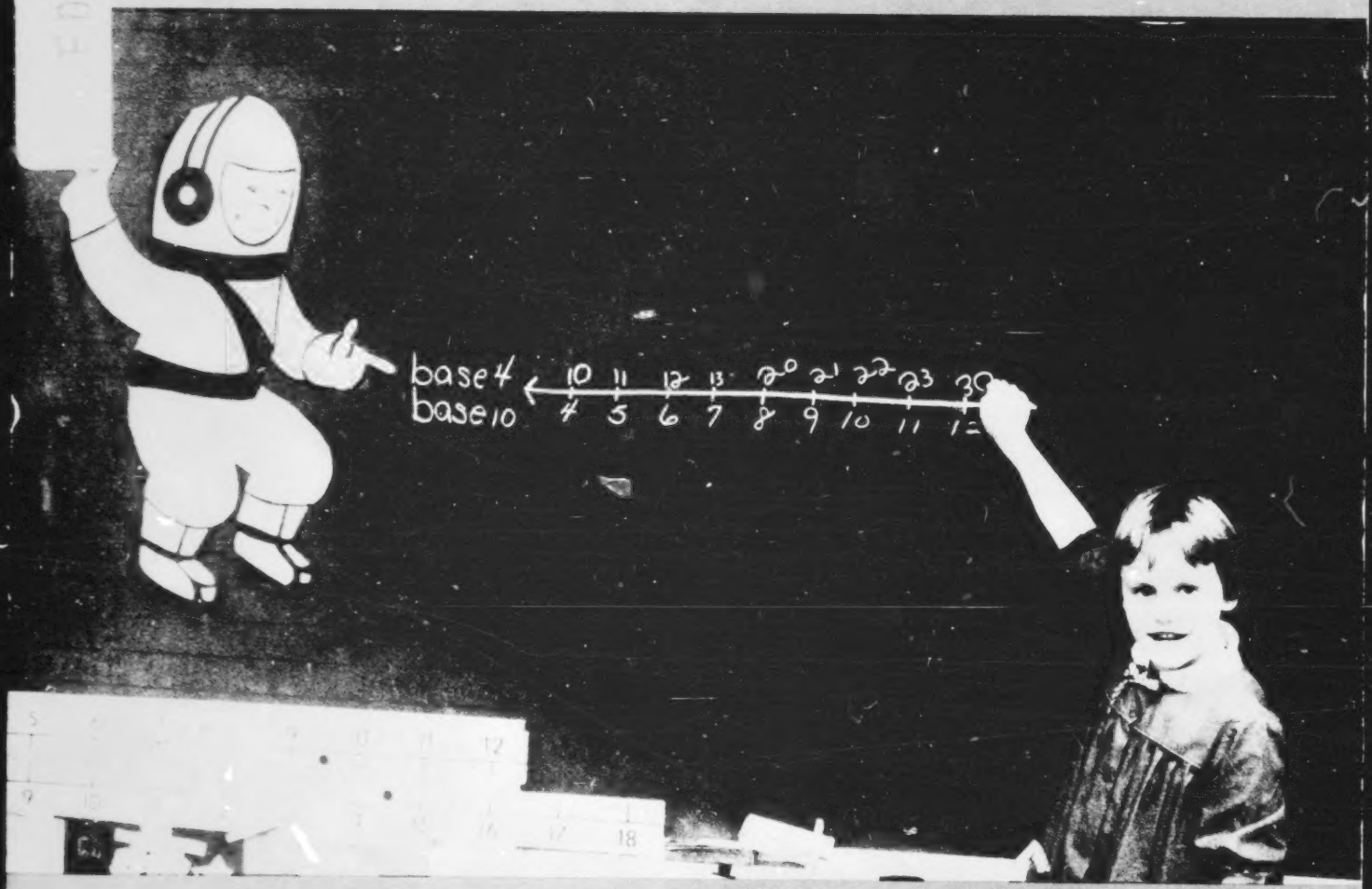

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MINNEMAST

UNIT

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

SE 021 221

MINNEMAST COORDINATED MATHEMATICS–SCIENCE SERIES

KINDERGARTEN	1. WATCHING AND WONDERING
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	9. NUMBERS AND COUNTING
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	12. MEASUREMENT WITH REFERENCE UNITS
	13. INTERPRETATIONS OF ADDITION AND SUBTRACTION
	14. EXPLORING SYMMETRICAL PATTERNS
SECOND GRADE	15. INVESTIGATING SYSTEMS
	16. NUMBERS AND MEASURING
	17. INTRODUCING MULTIPLICATION AND DIVISION
	18. SCALING AND REPRESENTATION
	19. COMPARING CHANGES
	20. USING LARGER NUMBERS
	21. ANGLES AND SPACE
	22. PARTS AND PIECES
THIRD GRADE	23. CONDITIONS AFFECTING LIFE
	24. CHANGE AND CALCULATIONS
	25. MULTIPLICATION AND MOTION
	26. WHAT ARE THINGS MADE OF?
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NUMBERS & MEASURING

LEARNING WITH TOR

UNIT 16

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT
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NUMBERS & MEASURING

This unit was developed by MINNEMAST on the basis of experiences of the many teachers who taught an earlier version in their classrooms.

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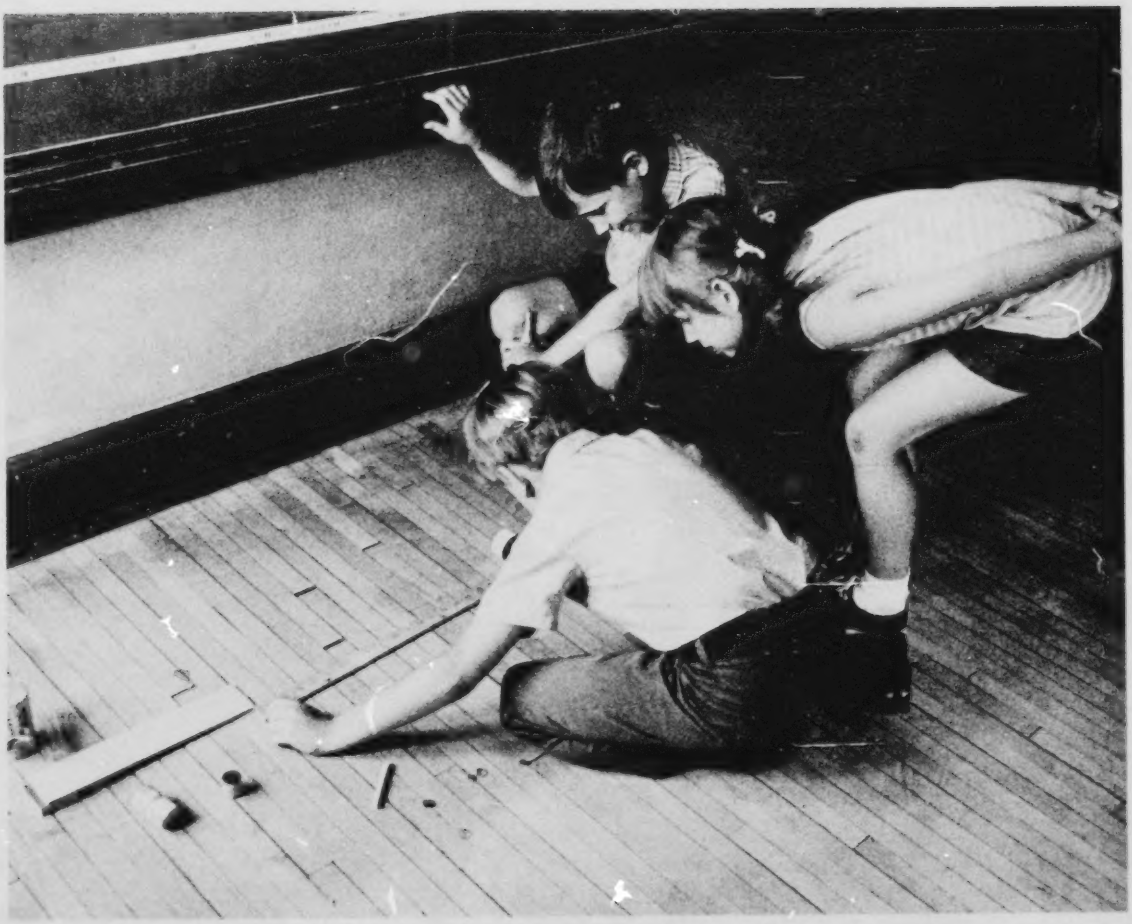

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Complete List of Materials for Unit 16

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
30	** Student Manuals	
1	** drawing of Tor	1,14
5	flat, rectangular objects, 2 of the same length	1
30	* MINNEMAST rulers	1,2,3
30	scissors	1
1	number line, 0-100 (from addition slide rule)	2,3,7,9
1	roll of masking tape	2,4
3-4	circles, squares or strips of paper or flannel	3
10	*** sets of Minnebars	3,14,20
1	*** thin, square property block	3
1	tape measure or *yardstick	3,8
30	*** addition slide rules	3,7,9,28
5	pieces of chalk: white, red, blue, yellow, orange	3,8
6	sets of 5 cylinders: 1 six-oz. orange juice can; 1 Dry cell; *1 tube from MINNEMAST viscosity kit; 1 large, thick circular property block; 1 small, thick circular property block	3,4
1	large cylinder, with length of *yarn longer than its circumference	4
25	* 10" lengths of yarn	4
30	* rulers	4,8,13,20
5	pens or crayons to mark yarn	4
1	United States flag	5
2000	* counters	6,12,13,14
1	flannel board	6,14
30	boxes of crayons	6,8
30	red pens or pencils	7,24
10	large books	8
30	* desk number lines, 0-25	8
5	* abaci	10,11,13,16

1 each	transparencies of Worksheets 26, 29, 35 (optional)	11
150	*souffle cups (small paper cups)	12
20	trays or boxes large enough to hold 10 paper cups	12
5-10	pictures of Egypt and Rome (optional)	17,18
2-3	*sets: Ping-Pong ball, rubber ball, ball of clay	19
2-3	sets: brick, *hard block, *soft block, closed box of similar size, etc.	19
2-3	sets: irregular stone, papier-mache lump, *styrofoam	19
2-3	sets: 3 cups containing materials of 3 different densities	19
30	sets: *paper clip, crayon, reader, scissors	19
110	*paper cups	20,21,22,24
1	seesaw (or 6' x 8" x 2" plank balanced on a brick)	20
10	beam balances made from *balance kit and *Tinker toy special set (see list on p. 110)	21,22,24,25
10	*rubber balls	22
2-3 each	scissors; *Ping-Pong balls	22
15 each	*1" paper clips, *corks	22,24
10	*2" paper clips	22,24
1 each	1" piece of chalk, 1" rubber eraser	22
1	scale to weigh children	23
10	2" x 14" strips of oak tag	24
10	rubber or elastic strings	24
10 each	8" Tinker toy rod, 3-1/4" Tinker toy rod, cylindrical connector	24
1	birthday candle, matches	25
10-12	objects, e.g., ice cubes, cotton, water, apple, sand, eggs, spring or elastic, clay, milk, pencil, chalk or crayon, bar of soap	25
30	**number lines, -15 to +15	27
	coins	28

*kit items as well as

**printed materials available from
Minnemath Center, 720 Washington Ave. S.E., Mpls., Minn. 55455

***available from The Judy Company,
310 North Second Street, Minneapolis, Minnesota 55401

INTRODUCTION

This unit is designed:

- To improve the child's ability to make quantitative measurements by giving him practice in measuring the properties of length and weight.

- To introduce the idea that measurements vary because of human error and imprecise measuring devices.

- To enlarge the child's comprehension of the real number system by extending his knowledge of numbers to 999, to fractions, and to negative integers.

- To explain the structure of the decimal place-value notation.

- To provide experience in calculating with denominate quantities (7 inches, 6 feet, 10 cents).

At this point in the MINNEMAST program, the child has had experience in comparing objects on the basis of simple properties and has started to study the real number system. In this unit we want him to learn to measure more accurately, using quantitative scales on rulers and balances. We also want him to learn more about the number system by examining some of the ways numbers are represented.

Previously the child made rather rough estimates of the properties of objects by comparing them two at a time (binary comparison). When determining the lengths of objects, he lined them up and arranged them in order by eye. When comparing weights of different objects, he judged their heaviness by hefting them in his hands. In this unit the child discovers that these simple binary comparisons do not give very accurate determinations of properties. He finds out that a ruler, or any such quantitative measuring device, gives more precise measurements. He also discovers the advantages to himself and others of this improved description of a property.

Length and weight are the two properties used to illustrate the need for quantitative measurement. The idea is that simple binary comparisons and subjective judgment do not

carefully describe certain kinds of properties. A more quantitative method is needed. The whole process of measurement is an attempt to extend the precision and accuracy of the human senses. Our eyes can judge length, but with a ruler we can measure it more precisely.

Measurement, then, is a tool we use for making more accurate and objective observations of the world. The better we can describe and analyze a thing, the better chance we have of understanding it, and measurement enables us to be quantitative in our descriptions. Thus measurement is a fundamental part of the scientific process, and the emphasis in this unit is on measurement rather than on length or weight per se.

In performing measurements, we always find a certain amount of variation in the results. Different people may get different answers, and no instrument is perfectly accurate. Therefore, in this unit we take up the subject of error and variation in measurement so that the child will have early experiences with the practical problems of making measurements.

In enlarging the child's view of the real number system, numbers up to 999 are introduced. Fractions and negative numbers are considered in a very rudimentary way, as are other mathematical ideas (the value of coins, the Egyptian method of representing numbers, etc.). Many of these extensions of the child's view of mathematics have important practical applications and will be emphasized in other units. But the emphasis here is on enlarging the meaning of the number concept. There is no wish to minimize, for example, the significance of fractions -- so essential in many kinds of calculations and measurements. But more important at this time than the child's ability to manipulate fractions is his realization that negative numbers, fractions, and numbers up to 999 (as well as the digits from 0 to 99 with which he is already familiar) are all part of the same set of real numbers.

Similarly, the base-four system of numeration is introduced only to clarify the decimal place-value system of notation we use for representing numbers. No emphasis at all should be placed on memorizing the base-four system. Also, all considerations of the subject, "number vs. numeral," are included only to help the children distinguish a number from the various

ways it can be represented. Some ways are useful for calculations, others for representing fractions, and still others for studying the properties of the numbers. It is precisely this flexibility of representation which needs emphasis. Once a child understands the concept and structure of numbers as distinct from the way they are represented, he can choose whichever symbols or systems of numeration best suit his purpose.

NOTES ON TEACHING THE UNIT

This coordinated unit should be taught during two class periods each day, and be completed in four or five weeks. However, you may wish to begin this unit before completing Unit 15, Investigating Systems. In that case it is suggested that you start teaching this unit in parallel with Unit 15, at any point you find convenient.

The first part of a continuing story, "Tor," appears at the beginning of Lesson 1. Tor is a very inquisitive little man from outer space. Teachers who have tried out this unit found the story highly motivating to the children. Children not only enjoyed pretending they were answering Tor's questions, but seemed to find it easier to explain things to him than to answer direct questions. You may use the Tor device as much as you see fit. For your convenience, a black and white figure of Tor is provided in the printed materials. This can be colored and mounted on cardboard by you or the children; or you may choose to draw your own figure of Tor or use one the children draw. In any case, make sure the figure has two-digit hands (only one thumb and one finger on each hand), as this is useful in introducing the lesson on base four.

Questions may arise about what "weight" is. To avoid long departures from the main subject of measurement, you can explain to the children -- quite simply -- that weight is the property of heaviness that all objects possess. We know that we are somewhat sensitive to this property because we can feel the weight of objects by holding them in our hands. Should some children be persistently curious about weight, they should not be discouraged, but should be told that the weight of an object is the pull on that object by the earth.

SECTION 1 MEASURING AND ORDERING

PURPOSE

- To review the proper use of the greater than (>), less than (<), equal to (=), and appears to be the same as ($\doteq$) symbols.
- To help children see the advantage of using different units of measurement.
- To give practice in the use of measuring tools.
- To introduce the use of fractional units in measurement and to give practice in the addition of half units.

COMMENTARY

This section reviews and extends earlier measuring experiences and emphasizes determining length and recording it. The records then are used as a basis for review and practice in arithmetic.

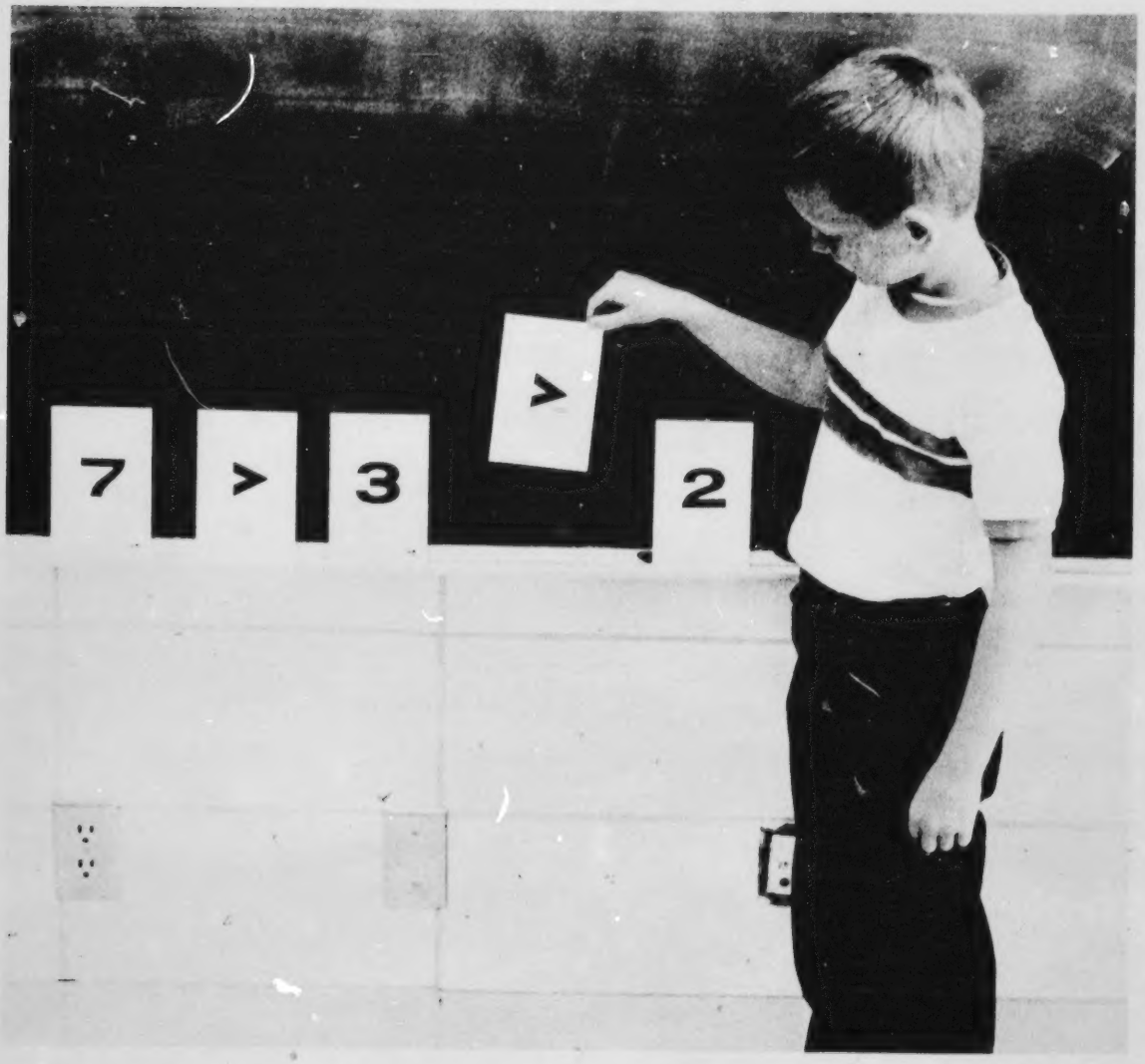
The section consists of four lessons, each requiring about two class periods. A story is read to the children to introduce the activities of Lesson 1. The activities include a review of the symbols listed above, ordering by length, and counting by ones. Measurements in this lesson are made with the centimeter scale.

In Lesson 2 the symbols are used to record comparison of numbers. The children use rulers with an inch scale to measure objects to the nearest inch or half inch. This information is plotted on a number line.

An experiment in Lesson 3 involves using ordinal numbers (first, second, third, etc.), measuring distances, and recording data. The number line is used for the addition of half units of measurement.

Lesson 4 introduces two new measurements that can be made on circular objects. The children have opportunities to investigate the relationship between circumference and diameter.

Measurements are plotted on the number line and more experience with whole and half units is given.



The boy in the photograph is using numeral and symbol cards to show that 7 is greater than 3 and 3 is greater than 2. If you wish to use cards in this way, they are easy to make; or you can borrow them from the first-grade supplies.

Lesson 1: A REVIEW OF ORDERING

A story introduces the activities of this lesson, which includes a review of the use of symbols, ordering by length, and counting. As in Unit 12, Measurement with Reference Units, length is defined as the measure of the longest dimension of an object.

Activity A reviews direct comparing of lengths and recording of the comparisons. You may want to remind the children that some objects can be compared for length, not only by lining up one end of each, but also by stacking objects appropriately -- the longest at the bottom. Try to guide the discussion so that the use of the symbols ($>$, $<$, $\doteq$) occurs naturally when the children are comparing the lengths of the various objects.

For Activities B and C, Worksheets 1-4 give individual practice in ordering the lengths of objects in two ways: by direct comparing and by measuring. Remind the children of the special ways the equality and approximation symbols are used.

The equality symbol ($=$) should be used to state the number of objects in a set of discrete objects, because the number is known to be precisely 3 and not 2 or 4. Example:

$$N_A = 3$$

This says, "The number of objects in Set A is exactly 3."

The approximation symbol ($\doteq$) should be used to state a measurement, because measurements are not precise. A measurement of 3 inches may be 2.999 or 3.001. Example:

$$L_A \doteq 3 \text{ inches}$$

This says, "The length of object A appears to be about 3 inches."

Activity D gives the children a review of the counting numbers from 0 to 99 (Worksheet 5).

MATERIALS

- story, "Tor," provided below
- drawing of Tor, provided in the printed materials
- 5 flat rectangular objects (boxes, rulers, books, etc.) -- 2 of which are apparently of the same length
- MINNEMAST rulers, 1 per child
- Worksheets 1-5 in Student Manuals
- scissors for each child

PROCEDURE

Read the story, "Tor," to the children. Later, perhaps during art or free time, they can help you color and mount on cardboard the figure of Tor provided in the printed materials. The figure (or another drawn by you or a child) can be placed on the chalk tray during the activities.

TOR

The children in the second grade were sitting at their desks, quietly reading. Miss Johnson, their teacher, suddenly realized that the room was much too warm. Speaking softly to one of her students, she said, "Tommy, it's such a beautiful day, almost like summer. Would you open the windows, please?"

As Tommy was doing this, he looked up and saw a streak of bright white light high in the sky. Then the light came nearer. It looked more red than white. The streak seemed to point toward the school yard just below the windows.

"Miss Johnson," Tommy called out, "something's coming down out of the sky. I think it's going to land on our playground!"

Everyone rushed pell-mell to the windows. All hearts pounded with excitement as the red streak slowed down. Then it gave off green and purple puffs of smoke. Then it went beeple, beeple, plop, and landed -- without another sound -- right in the center of the school yard.

"It's a space ship! I just know it's a space ship!" Tommy exclaimed.

"My goodness, Tommy, I think you're right!" Miss Johnson sounded just as thrilled as the children.

"It looks just like a great big rubber ball!" Susan shouted.

"And it has red and white stripes like peppermint candy!" Mary said.

Tommy pointed to the space ship. "Oh, oh, a door is opening!"

And sure enough, one of the red stripes slid open, a ladder popped out, and a little man in a space suit climbed down the ladder to the ground. He looked up and waved to the children and Miss Johnson. Everyone waved back.

Then the little man raised his visor and called out, "Is this the Earth?" and the children all shouted back, "Yes, yes, it is!"

"Is this a school?"

"Yes, yes!"

"Good! I was hoping it was, but I couldn't be sure. May I come up for a visit?"

The children surrounded Miss Johnson, begging her to say yes. "He's very friendly -- he even waved to us," they reminded her.

Miss Johnson smiled. Then, leaning from the window a little, she called out, "We'll come down and show you the way to our room."

"Thank you very much, but that won't be necessary," the space man said. And before anyone had a chance to decide what to do next, he reached in a pocket of his space suit and took out a can.

"That looks like some kind of a spray can," Joe said.

And that is just what it turned out to be! The children and their teacher stared in amazement as the space man pointed the can at the ground and began to spray. As he sprayed down, he rose up, up, up, in the air. When he was just above the window sills, he sprayed away from the school and flew backwards through an open window right into the classroom. Then he stopped spraying and floated gently to the floor.

Now the children clustered around the space man. They admired his shiny blue suit and his bright red boots. When he took off his helmet, they were delighted to see a round, friendly face with rosy cheeks, and hair brushed to a neat point on top of his head. They noticed his hands -- he had hands that looked something like mittens, with just a thumb and one finger on each. And Tommy noticed that the space man was much shorter than anyone in the room, but, of course, he didn't say anything about that.

"Good morning, Earth people," the space man said. "I am Tor, from Titan. Titan is the largest moon of the planet Saturn. I can hardly believe that I have really found the Earth. Are you sure this is the Earth and that this is truly an Earth school?"

"Yes, Tor," the teacher answered. "This is a class in the second grade. I am Miss Johnson and these are my students."

"That's wonderful!" Tor said. "I could not have selected a better place to land. On Titan, many of us have been listening to your radio programs and so we have learned to speak your language. Now I have been sent to learn how to read all your written symbols, especially those you use in mathematics and science. Can I learn such things here?"

"Yes, we study all kinds of things like that," Miss Johnson said. "Today we are going to review some symbols we use when we compare the lengths of objects."

"Well, could I stay in this class and learn along with your students?" Tor asked.

Miss Johnson said, "Well, what do you think, children? Shall we let Tor stay and learn along with us? Will you be willing to explain things to him and try to find answers to his questions?"

Together the children all shouted, "Oh, yes, yes!" and "Tor, please stay!"

Tor practically jumped up and down for joy. "Oh, how lucky I am!" he exclaimed. "People on Titan said you might not like me -- that you might not be friendly at all. I was so afraid."

Then, to show how friendly they were, the children all surrounded Tor again and started asking questions. The questions came out in such a rush that Tor could hardly keep track of them.

Tommy asked, "How long did it take you to get here, Tor?" and Joe wanted to know how many miles Tor had traveled. Sally asked, "Where are you going to stay?" and Mary wanted to know what he liked to eat.

Miss Johnson rapped a pencil on the desk, for order. When it was quiet in the room, Tor turned to Tommy. "It took a very long time for me to get here and I can tell you how much that is in my time -- it's segura ziggeraties -- but I really don't know how to figure anything in your time, so that answer will have to wait until I learn." Turning then to Joe, Tor said, "When I learn your units of measurement, I'll be able to tell you how far I traveled, but I certainly can't do that now." To Sally and Mary, he said, "You don't have to worry about where I will stay or what I will eat, because I will live in my space ship, where I have everything that I need. After school you are all invited to come and inspect my ship, but right now I am very eager to start learning, because there's so very much I don't know."

Miss Johnson offered Tor her chair. Tor looked it over.

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"It's much too tall for me, Miss Johnson," he said. "It would not be comfortable."

Miss Johnson sent Tommy to the kindergarten to fetch a smaller chair, but when Tor saw it, he said, "This chair is less tall than your chair, Miss Johnson, but it's still too tall for me."

"My," Sally said, "you are fussier than Goldilocks in the story of the three bears," but Tor wasn't listening. He sprayed himself out to his ship and back again, and when he came back he was carrying a little chair that he said was "just right." He put it beside Tommy's seat, announced, "I'm ready now," and sat down.

When Miss Johnson saw that Tor was comfortable, she put some objects on her desk and said, "Let's begin by showing our visitor how we make comparisons," and the lesson began.

.....

Activity A

Place five flat rectangular objects where the children can see them. (Remember that two of the objects should be of the same apparent length.) Ask one child to pick up any two objects (e.g., a book and a ruler) and hold them up for the class to see. The following questions assume that Tor is learning along with the children. Whether you use this pretense or not, guide the discussion to the property of length.

CAN YOU TELL TOR WHICH OF THE TWO OBJECTS HAS THE GREATER LENGTH? (The ruler.)

SHOW TOR HOW WE CHECK THE COMPARISON. (We place one object beside the other.)

HOW COULD WE RECORD THIS COMPARISON? (We could write, "The length of the ruler is greater than the length of the book.")

CAN ANYONE SHOW TOR A SHORTER WAY TO RECORD THE COMPARISON?

Write $L_R > L_B$ on the chalkboard.

CAN YOU TELL TOR WHAT THE L STANDS FOR? (Length.)

WHAT THE R STANDS FOR? (Ruler.)

WHAT THE B STANDS FOR? (Book.)

HOW DO WE READ THE SYMBOL? (Is greater than.)

NOW PLEASE READ THE WHOLE COMPARISON SENTENCE FOR TOR. (The length of the ruler is greater than the length of the book.)

Have another child compare and then record the relative lengths of two other objects. Repeat this several times until the greater than (>), the less than (<) and the approximation ($\doteq$) symbols have all been used and discussed.

The children may enjoy recording a comparison of the lengths of the three chairs in the Tor story:

$$L_J > L_K > L_T$$

(The length of Miss Johnson's chair is greater than the length of the kindergarten chair, which is greater than the length of Tor's chair.)

They could also reverse this ordering, using the < symbol:

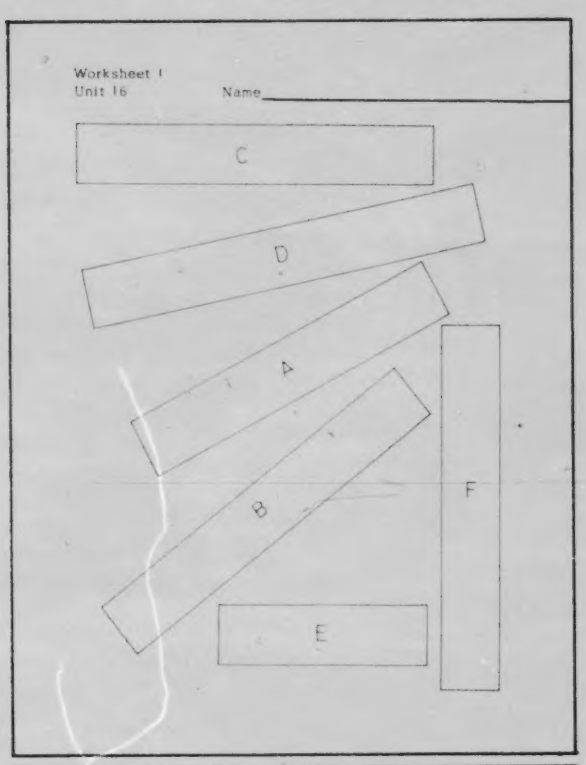
$$L_T < L_K < L_J$$

(The length of Tor's chair is less than the length of the kindergarten chair, which is less than the length of Miss Johnson's chair.)

If there are children in your class who have not had MINNEMAST in first grade, they probably will need additional practice in using the symbols before going on to the next activity. Numeral and symbol cards may prove useful here (see photograph on page 5).

Activity B

Distribute the student manuals and have each child carefully cut out the six objects on Worksheet 1.



Worksheet 2
Unit 16 Name

Compare the lengths of your paper strips, two at a time. In the blanks, write $<$, $>$ or $\doteq$.

l_A ___ l_B

l_C ___ l_D

l_E ___ l_F

l_A ___ l_F

l_B ___ l_E

l_D ___ l_C

Order all your paper strips by length on your desk. Fill in the name of each strip and $<$, $>$ or $\doteq$.

$l_E < l_A < l_C \doteq l_F < l_B < l_D$

(or F) (or C)

Tell the children they are to compare two of these paper strips at a time for the property of length, then fill in the proper symbols on Worksheet 2. Have one child make the first binary comparison of strips A and B, and record it on the board for the class and Tor. Then let the children work individually on the five other binary comparisons.

When they have completed the six statements, ask them to arrange their six paper strips in order on their desks -- the shortest at the left. Then explain that in the general ordering on the last line of Worksheet 2, they are to fill in first the letter symbols or names of their strips (A for strip A, etc.) and then the correct relation symbols.

Have the children save their paper strips for use in Activity C and for Lesson 2.

Activity C

In this activity the children use their paper strips, a ruler with a centimeter scale, and two worksheets. Begin with a review of the proper uses of the equality (=) and approximation ($\doteq$) symbols (for Tor, if you like).

Ask six children to come to the front of the room. Tell three to stand at the left and three at the right. Name one group "Set A" and the other "Set B."

HOW MANY MEMBERS DOES SET A HAVE? (3.)

Write $N_A = 3$ on the chalk board.

HOW MANY MEMBERS DOES SET B HAVE? (3.)

Write $N_B = 3$ on the chalk board.

IS THE NUMBER OF MEMBERS IN SET A EQUAL TO THE NUMBER OF MEMBERS IN SET B? (Yes.)

Write $N_A = N_B = 3$ on the chalk board.

NOW TELL TOR WHEN WE USE THE EQUAL-SIGN. (When two sets have an equal number of members; when we can count the members and see the equality.)

Add a member to Set A, then ask the same questions again. This time the final answer is that we cannot use the equal-sign because 4 is greater than 3. ($4 > 3$.)

Since $N_A = 4$ and $N_B = 3$, then $N_A > N_B$.

DO WE USE THE EQUAL-SIGN WHEN WE RECORD MEASUREMENTS OF LENGTH? (No, because measurements are not exact.)

TELL TOR WHAT SIGN WE USE. (We use $\doteq$, the "appears to be the same as" symbol.)

Worksheet 3
Unit 16 Name_____

Measure each strip with a centimeter ruler and write the lengths in the blanks.

$L_A \doteq$ 14 cm

$L_B \doteq$ 16 cm

$L_C \doteq$ 15 cm

$L_D \doteq$ 17 cm

$L_E \doteq$ 9 cm

$L_F \doteq$ 15 cm

Order the measurements here. Start with the smallest.

9 cm < 14 cm < 15 cm $\doteq$ 15 cm < 16 cm < 17 cm

Now tell the children to show Tor how they can check their previous ordering of the six paper strips. Have them use the centimeter scale on their rulers to measure each paper strip and then record the length of each strip with a numeral on Worksheet 3. Remind the children to line up one end of each strip with zero (which is not at the very end of a MINNEMAST ruler).

On the last line the children fill in the numerals representing the measurements -- starting with the smallest numeral. One (15) is already given in this ordering.

ARE YOUR NUMERALS IN COUNTING ORDER?

Check to see that the children place the smallest numeral on the left, the largest on the right.

Ask the children to compare this new ordering with the ordering on Worksheet 2. Ask one child to copy the first ordering on the chalk board and another child to put the second ordering directly below it.

Have the class check to see whether the two orderings are correct. Remind the children to use the list of binary comparisons ($L_A \doteq$ 14 cm, etc.) on Worksheet 3 as a reference.

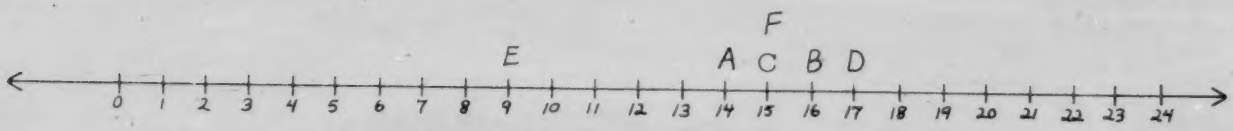
On Worksheet 4 the children record on the number line the name of each strip (its letter) at the numeral which represents the length of the strip. The length of C is already plotted for them.

Worksheet 4
Unit 16 Name_____

How many centimeters long is your A strip?
Mark an A at that point on this number line.

How many centimeters long is your B strip?
Mark a B at that point on this number line.

Do the same for the rest of your strips.



Activity D

This is a review of the counting numbers from 0 to 99. The children fill in the missing numerals on Worksheet 5.

Worksheet 5
Unit 16 Name_____

Start at 0 and work this way ——→ across each row of squares. Fill each blank square with the next counting number.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Point out that some of the numerals the children put in order on Worksheet 3 are found on this worksheet, with some others. Read the directions with the class before the children start writing in the numerals. Remind them that each numeral is one more or one less than its neighbor. Tell them that they may refer to the number line if they wish.

Save the completed worksheets for correction and review in Lesson 2.

Lesson 2: MEASURING TO THE NEAREST HALF INCH

This lesson gives a review of the ordering of numerals and then introduces the measurement of length to the nearest half inch. The children record their measurements and plot them on number lines.

Earlier units emphasized the fact that measurements can never be perfectly accurate by having the children express their results in two ways:

(1) They measured within an interval of length and recorded the measurement as

$6 \text{ in.} < L_B < 7 \text{ in.}$ (Six inches is less than the length of object B, and the length of object B is less than seven inches.)

(2) They made a simple approximation and recorded it as

$L_B \doteq 6 \text{ in.}$ (The length of object B appears to be about 6 inches.)

In this lesson the children again use the approximation symbol to express measurements to the nearest half inch:

$L_B \doteq 6 \text{ in.}$

$L_A \doteq 5\frac{1}{2} \text{ in.}$

Some children may need help in deciding whether a length is $6\frac{1}{2}$ or 7 inches long. Guide them to choose the closer reading. When a measurement appears to be midway between two readings, let the child choose the reading he prefers. Experience like this reinforces the idea that measurements are only approximations.

MATERIALS

- number line (0-100) taped on the chalk board

-- for each child --

- Worksheets 5-7
- MINNEMAST ruler
- paper strips A, C, D and E from Lesson 1, or a set of four Minnebars (such as 1 pink, 1 gray, 1 purple, and 1 green) labeled A, C, D and E respectively
- 1 object of the child's choice, less than 7" in length

Activity A

Have the children review and correct Worksheet 5 for Tor.

Activity B

Worksheet 6
Unit 16 Name_____

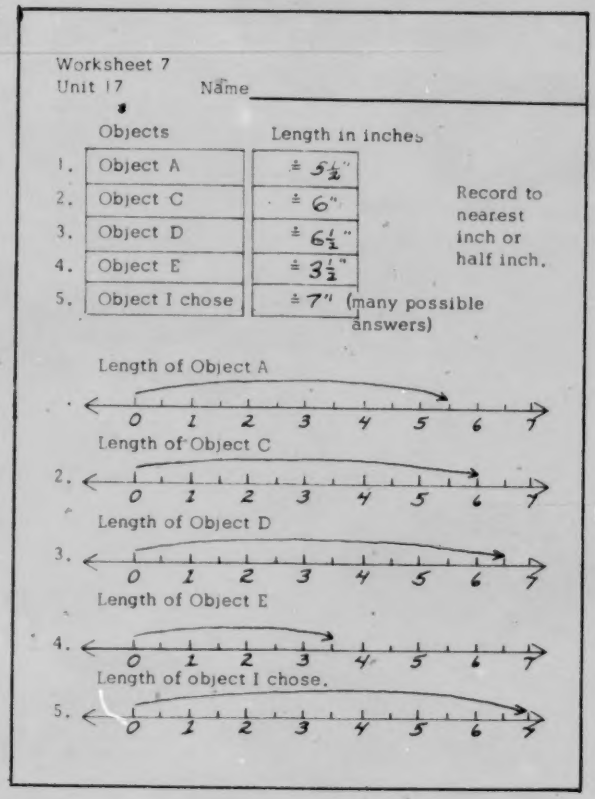
Fill in each blank with >, < or =. Look at the number line or at Worksheet 5 if you wish.

9 > 6	1 < 4
5 > 2	3 > 0
3 = 3	44 > 27
2 > 1	96 > 69
10 > 4	84 > 37
23 < 49	24 = 24
60 > 32	15 < 32

Ask the children to turn to Worksheet 6, and explain the instructions to them. Do one or two examples as a class exercise. Suggest that the children may use corrected Worksheet 5 and the number line on the board for reference. Ask the children to complete the rest of Worksheet 6 individually. As soon as possible after completion, check the answers with the class.

Activity C

Have the children take out their MINNEMAST rulers. See if they can tell Tor that the marks between the numerals on the inch scale indicate half inches. Then explain that they are going to measure the length of objects to the nearest half inch.


Worksheet 7
Unit 17 Name _____

	Objects	Length in inches
1.	Object A	≐ $5\frac{1}{2}$"
2.	Object C	≐ 6"
3.	Object D	≐ $6\frac{1}{2}$"
4.	Object E	≐ $3\frac{1}{2}$"
5.	Object I chose	≐ 7" (many possible answers)

Record to nearest inch or half inch.

Length of Object A
0 1 2 3 4 5 6 7
2. Length of Object C
0 1 2 3 4 5 6 7
3. Length of Object D
0 1 2 3 4 5 6 7
4. Length of Object E
0 1 2 3 4 5 6 7
5. Length of object I chose.
0 1 2 3 4 5 6 7

Have the children measure and record on Worksheet 7 the lengths of paper strips A, C, D and E (or Minnebars so labeled) and an object of their own choice -- not longer than seven inches.

Next have the children use number line 1 to plot the length of object A, and number line 2 to plot the length of object C. Then, before plotting the length of object D, ask them to write in the numerals on number line 3. When the children have done this, direct them to mark off the inches and half inches on number lines 4 and 5, and to number these two lines. (A few children may need help in numbering or marking the number lines.) Then they are to plot the last two lengths.

Some children may feel frustrated at being required to measure only to the nearest half inch, when they are capable of measuring quarter inches. Since MINNEMAST rulers do not indicate quarter inches, these children should be permitted to work with other rulers and record their measurements on plain sheets of paper, but only after they have completed Worksheet 7 according to directions.

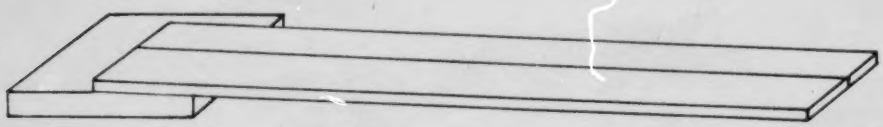
Save the completed worksheet for use in the next lesson.

Lesson 3: MEASURING AND ADDING FRACTIONAL UNITS

This lesson continues the work with fractional units, reviews the idea that two halves combine to make a whole, and gives practice in the use of ordinal numbers (first, second, third, etc.).

Activities include a worksheet on which lengths measured to the nearest half inch are added, as well as some sets of objects with whole and half members. An experiment where objects roll from an inclined plane to a level surface provides further opportunities for measuring to the nearest half inch.

The inclined plane can be constructed very simply with a property block and two rulers, as shown:



MATERIALS

- a few paper or flannel circles, squares and strips
- number line, 0-6, of very large units drawn on chalk board
- 1 number line tape (to 100)
- Minnebars
- 2 MINNEMAST rulers
- 1 thin square property block
- 1 tape measure or yardstick
- 1 addition slide rule
- 1 piece of white chalk, 1 red, 1 blue
- 4 or 5 cylindrical objects, such as: 6 oz. juice can, dry cell (D), empty viscosity tube without cover (from first-grade kit), large and small circular property blocks, crayons or pencils, small cars (size of Matchbox Series) if children can bring them

- list of ordinal numbers on chalk board, as follows:

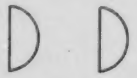
First	first	1st
Second	second	2nd
Third	third	3rd
Fourth	fourth	4th
Fifth	fifth	5th

- Worksheet 8

PROCEDURE

Activity A

To prepare the children for their individual work in adding fractional units (halves), review with them the idea that a whole is made up of two equal parts and that each of these two equal parts is one half of the whole. Demonstrate this with drawings on the board or by cutting paper strips, circles and squares in half, and fitting the halves together again. Pretend that a paper circle is a cookie, cut it in half and line up the halves like this:



IF THESE TWO HALVES ARE TWO EQUAL PARTS OF A COOKIE, HOW MANY COOKIES WOULD I HAVE IF I ADDED THE TWO TOGETHER? ($\frac{1}{2} + \frac{1}{2} = 1$.)

Have a child check this by turning the second half "cookie" to fit the first:

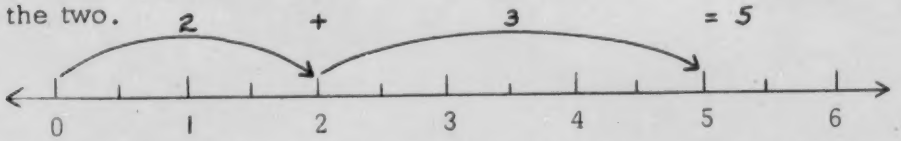
Give as many similar examples as seem necessary for the children to understand that $\frac{1}{2} + \frac{1}{2} = 1$.

Now pretend that 2 paper squares and 3 half squares are cookies and half cookies, and arrange them like this:

HOW MANY COOKIES WOULD I HAVE IF I ADDED ALL THESE WHOLE AND HALF COOKIES?
($1 + \frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{2} = 3\frac{1}{2}$.)

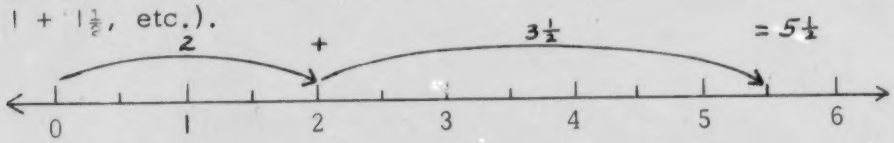
If children have trouble adding these, encourage them to fit as many halves into wholes as possible, and then count.

To review the addition of whole units of length, use a number line of six large units on the chalk board. In showing how to add 2 and 3, for example, draw first a curve from 0 to 2, then a curve from 2 to 5. The children should be able to see that the numeral at the end of the second curve is the sum of the two.

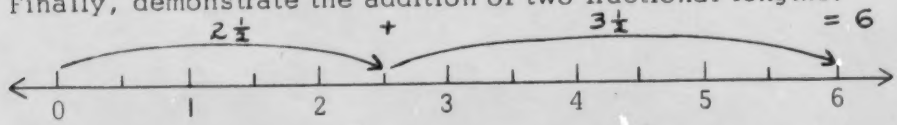


Have the children demonstrate by marking off the solutions to such problems as 4 + 1, 3 + 2, 3 + 3, etc.

Now use the same number line to have the class do a few examples where they add one length that is a whole number of units to a length that has a fraction ($2 + 3\frac{1}{2}$, $1\frac{1}{2} + 3$, $1 + 1\frac{1}{2}$, etc.).



Finally, demonstrate the addition of two fractional lengths:

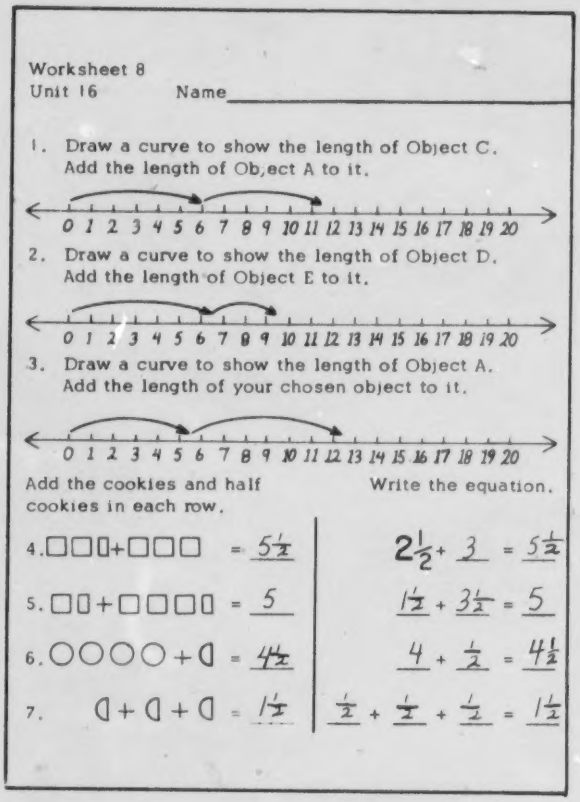


Have the children practice adding other fractional lengths on the number line until you feel sure they understand how to do it. For those having difficulty, provide pairs of Minnebars so that the children can measure each bar with a ruler, then place both together at the ruler's edge to read the result.

Activity B

Remind the children that on Worksheet 7 they recorded the lengths of five objects to the nearest half inch. On Worksheet 8 they will use number lines to add the numbers which represent these lengths. Point out that the worksheet is not wide enough to hold a number line of twenty inches, but the marks will <u>represent</u> inches. In the first three problems the

children will be adding the number of inches and half inches. In the last four problems, they will be adding objects and halves of objects. Read the directions on the worksheet and have the children follow along. When you think they understand the procedure, have them complete the worksheet individually, with as little help as possible. In the class discussion that follows, let each child check and correct his results. There may be a little variation in the answers to problems 1 and 2, because some children may choose a reading a half unit different from others, but the answers to problem 3 should vary greatly. Ask about this:



Worksheet 8
Unit 16 Name_____

1. Draw a curve to show the length of Object C.
Add the length of Object A to it.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

2. Draw a curve to show the length of Object D.
Add the length of Object E to it.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

3. Draw a curve to show the length of Object A.
Add the length of your chosen object to it.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Add the cookies and half cookies in each row. Write the equation.

4. □□▯+□□□ = $5\frac{1}{2}$ $2\frac{1}{2} + 3 = 5\frac{1}{2}$

5. □▯+□□□▯ = 5 $1\frac{1}{2} + 3\frac{1}{2} = 5$

6. ○○○○+◖ = $4\frac{1}{2}$ $4 + \frac{1}{2} = 4\frac{1}{2}$

7. ◖+◖+◖ = $1\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1\frac{1}{2}$

DO WE HAVE MANY DIFFERENT ANSWERS FOR PROBLEM 3? (Yes.)

WHY? (Because we chose many different objects to add to the length of object A.)

Some children may not see that the answer to problem 7 is $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1\frac{1}{2}$. If so, review the cutting and fitting together of the paper cookies. Elicit from the class that half an object remains half an object no matter how it is turned.

Activity C

This activity involves measuring the distances that cylindrical objects roll from an inclined slope and familiarizing the children with some of the ordinal numbers. The equipment consists of a long number line taped on the chalk board at a height where the children can easily reach it; two MINNEMAST rulers propped on a small, square property block; four or five cylindrical objects (marked A, B, C, D, etc.) to be used in each "race"; and one tape measure or yardstick. Since an object may roll six feet or more, even when gently released at the top of the slope, select a fairly large area where all children can see what happens. A location near the number line would probably be most convenient.

Introduce the activity by asking the children whether they have ever rolled toy cars downhill to see how far the cars would go. Say that today the class is going to roll some objects down a slope for the same reason.

Explain to the children that they are going to observe, measure and record the distance each object rolls after it comes off the slope. Tell them to think of the activity as a race to see which object will roll the farthest.

Then tell the children they will have a good opportunity to use some of the ordinal numbers they see on the chalk board. They can say that the object that goes the greatest distance is the winner -- it is <u>first</u>, another object will come out <u>second</u> in the amount of inches it travels, and so on.

For each race, assign four different children for these tasks:

1. To release the objects gently, one at a time, at the top of the slope.

2. To mark with chalk the spot on the floor where each object stops.

3. To measure to the nearest half inch the distance each object travels from the base of the slope.

4. To record each measurement on the number line, marking it with an A, a B, etc.

Have the children record with white chalk for the first race, red for the second, and blue for the third. Provide child 3 with a tape measure or yardstick. Help him if he has to add large numbers.

After each object has been rolled, and the distance marked, measured, and recorded, ask the children which object came in first, which second, etc. Use these ordinal numbers frequently and refer to them as ordinal numbers. The children will need to recognize the abbreviations (1st, 2nd, 3rd, 4th and 5th) for the worksheet in the next lesson, but they need not go beyond that now, as they will receive more practice with ordinal numbers in future lessons.

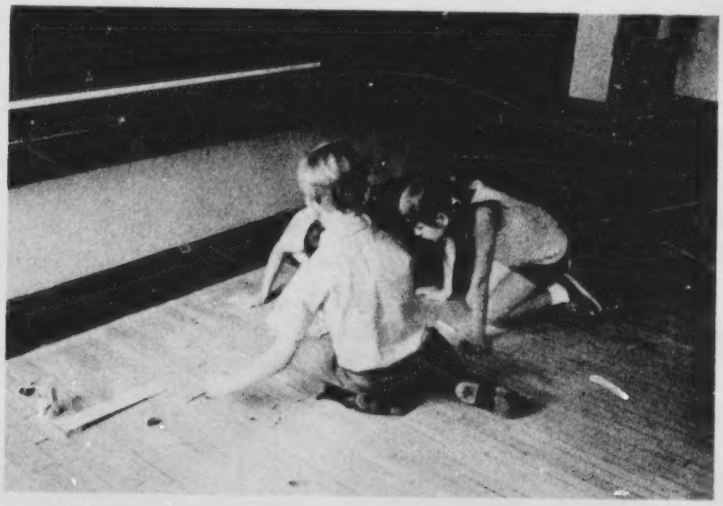
At the end of the first race, ask the children what would happen if the same experiment were repeated.

DO YOU THINK WE WOULD GET THE SAME RESULTS IF WE RACED THESE SAME OBJECTS AGAIN?

HOW CAN WE FIND OUT?

Erase the floor marks from the first race and have the class repeat the experiment with other participants. Ask whether the results are the same. Then ask whether the results would be the same for a third race, and let them conduct that.

If there is time, the children may enjoy racing the same objects from a slightly higher slope, or a different set of objects from the same slope, etc.



Lesson 4: DIAMETERS AND CIRCUMFERENCES

Measuring diameters and circumferences of cylindrical objects gives the children further practice in determining and recording lengths to the nearest half unit. The lesson also takes a first step toward developing in the children some appreciation of the relation between the diameter and the circumference of a circle.

In Activity A the children hold yarn around each of five cylinders, mark the yarn where it overlaps, then measure the marked segment of yarn to the nearest centimeter or half centimeter. They record these five measurements of circumference in Table A of Worksheet 9.

In Activity B the children draw an outline of one end of each cylinder and measure the diameters of these circular outlines with their rulers. They record -- to the nearest half centimeter -- the lengths of the five diameters in Table B of Worksheet 9. They also check the accuracy of the first diameter measurements by measuring another diameter of each circle.

Then each child selects the two measurements of just one object to plot on number lines. On a third number line he plots the length of the circumference of his selected object with a red crayon above the line, and repeated lengths of the diameter of the same object with a blue crayon below the line. He thus discovers that the circumference is approximately three times longer than the diameter of a circle (and/or cylinder).

For the purposes of this lesson, define diameter very simply as "the distance across a circle" and circumference as "the distance around a circle." Use the terms "diameter" and "circumference" as often as you can fit them in naturally, so that the children will begin to use them, too.

MATERIALS

- 1 rather large cylindrical object (such as a 3-lb. coffee can, canister or waste basket) and a piece of yarn slightly longer than its circumference
- Worksheet 9

-- for each group of five --

- 1 six-oz. juice can; 1 dry cell (D); 1 tube from viscosity kit; 1 large, thick, circular property block; 1 small, thick, circular property block. (In the order listed, label the objects: 1st, 2nd, 3rd, 4th, and 5th.)
- 25 pieces of yarn, each about 10" long
- 5 rulers with centimeter scale
- masking tape (optional)
- 5 crayons, soft lead pencils, or pens for marking the yarn

PROCEDURE

Activity A

Display a set of the five cylindrical objects that the groups of children are going to use, along with a ruler, a crayon or other marking tool, a sheet of paper, and a piece of yarn.

Explain that the five objects are cylindrically shaped, much like some of those the children used in the last lesson. Say that this time, instead of measuring how far the objects roll from a slope, the children are going to make measurements on the objects themselves.

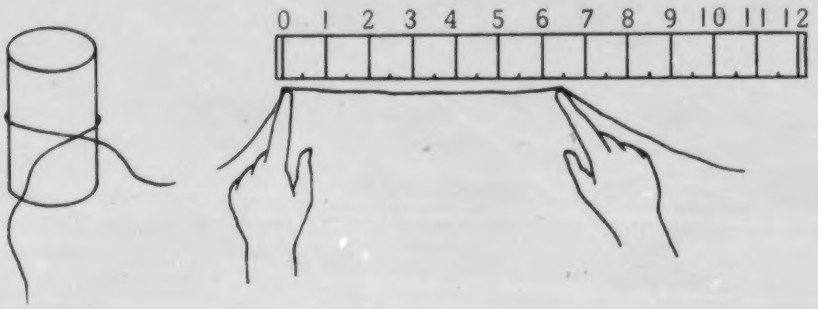
Pick up two objects from the set.

CAN ANYONE SHOW TOR HOW WE COULD COMPARE THE CIRCUMFERENCES -- THE DISTANCES AROUND THE CURVED SURFACES -- OF THESE TWO OBJECTS?

Have the children come to the front of the room and try out whatever methods of comparing they suggest. Some children may remember that in kindergarten or first grade they put string or paper around each circular object, cut these at the point or line of overlap, and then compared the lengths of the strings or the pieces of paper.

If no child uses the yarn to mark off the distance around an object and the ruler to provide units of measurement, suggest that this be tried. If necessary, give a demonstration

of the procedure: Select a cylinder that is not in the children's set (e.g., a coffee can) and wrap a piece of yarn around it. Mark the two parts of the yarn at the point of overlap, and then measure the marked segment with the ruler.



Have a few children try the method. Some may be dexterous enough to hold and measure a segment of yarn without marking it. Others may need to fasten the yarn to the cylinder with tape in order to do the marking. In any case, point out the importance of fitting the yarn closely around the object and of holding the yarn firmly in place until it is marked (or securing it firmly at the two appropriate places with fingers of both hands until it is measured).

Activity B

Have monitors distribute the sets of cylinders, yarn, etc. to the groups of children. Ask the children to take turns measuring the circumference of each object to the nearest half centimeter and to record the five measurements in Table A of Worksheet 9. The following remarks may be helpful:

> USE A NEW PIECE OF YARN FOR EACH OBJECT, IF YOU ARE GOING TO MARK THE YARN. MAKE TWO MARKS ON THE YARN WHERE IT OVERLAPS. MEASURE THE DISTANCE BETWEEN THE TWO MARKS TO THE NEAREST CENTIMETER OR HALF CENTIMETER. IF THERE ARE NO HALF CENTIMETER MARKS ON YOUR RULER, DECIDE FOR YOURSELVES WHERE THE HALF MARKS SHOULD BE.

If any member of a group is having difficulties, suggest that the other members assist him.

When all have finished filling in Table A of the worksheet, hold a discussion about the value of knowing how to measure the circumference of an object:

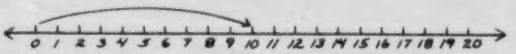
CAN YOU GIVE TOR ANY REASONS WHY IT MIGHT BE USEFUL TO KNOW HOW TO MEASURE THE CIRCUMFERENCE OF AN OBJECT?

Any child's reason is adequate for this discussion (finding a length of paper to cover an object, shopping for a new belt to surprise Daddy on his birthday, etc.).

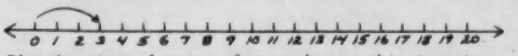
Worksheet 9
Unit 16 Name_____

A Object	Circumference		B Object	Diameter	
1st	18 cm	Juice can	1st	5½ cm	
2nd	11 cm	Dry cell	2nd	3 cm	
3rd	10 cm	Tube	3rd	3 cm	
4th	16½ cm	Large block	4th	5 cm	
5th	8½ cm	Small block	5th	2½ cm	

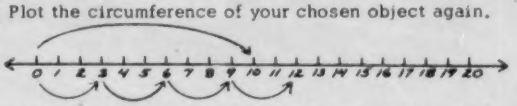
A Plot the circumference of your chosen object on number line A.



B Plot the diameter of chosen object on number line B.



C Plot the circumference of your chosen object again.

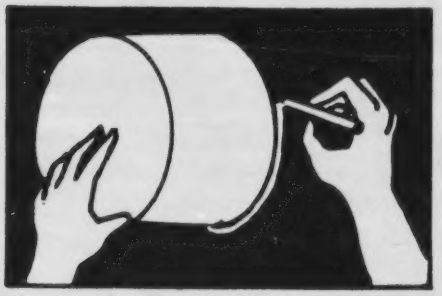


Now below the number line plot diameters of your chosen object over and over until you have just passed the end of the circumference.

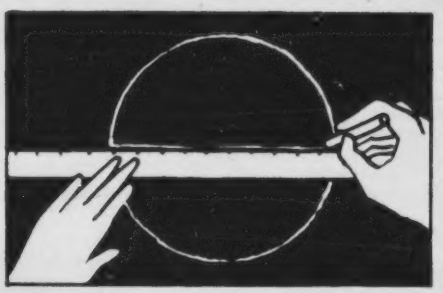
Jumps taken to come near circumference 3

Activity C

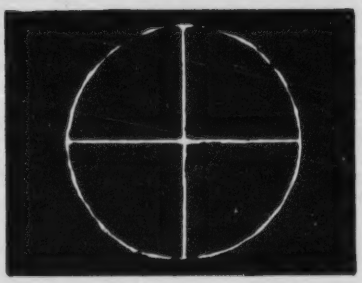
Show the children how to draw the outline of one end of a cylinder by holding a large object (such as a wastebasket) against the chalk board and drawing around it with a piece of chalk.



Then draw a straight line across the circle on the board and explain that this is one diameter (one line of greatest distance) of your circle. Measure this diameter with a yardstick or meterstick and record the measurement on the board.



Next draw a vertical diameter of the circle, and measure and record it. Draw, measure and record a few more diameters until the children notice that all your diameter measurements are the same.



Now explain to the children that they are to draw outlines of each cylinder (in the group's set) on the back of Worksheet 8. They are to pass the objects around, so that all can be working at the same time. But before giving an object to the next child, they must write the ordinal number of the object (1st, 2nd, etc.) inside its circular outline.

Ask the children to draw their circles carefully, pressing down firmly on each object and keeping the pencil point as close to the object as they can. Tell them not to crowd the circles together on the paper, but to leave room for recording some measurements.

When all have neatly drawn and suitably labeled the five circles, ask the children to draw one diameter straight across each circle (horizontally). Then have them measure each diameter to the nearest centimeter or half centimeter and record each measurement near the appropriate circle. Ask how this measurement could be checked. When the children suggest checking it by drawing and measuring another diameter for each circle, ask them to draw an up-and-down (vertical) diameter in each, and to measure and record these near their first records. Check the results:

> WHEN YOU MEASURED THE SECOND DIAMETER OF EACH CIRCLE, DID YOU GET THE SAME MEASUREMENT AS THE FIRST TIME?

If the two diameter measurements by the same child for the same circle vary by more than half a centimeter, check his paper to see whether either his circles or his diameters are too poorly drawn to be useful. If so, have him draw and measure others with your guidance. If his drawings are good enough, ask him to measure again more carefully. Children may have drawn circles of slightly varying diameter for the

same object so it is not necessary for all measurements in a group or in the class to agree, but each child should have -- for each of his own circles -- a fairly accurate measurement.

WHEN YOU ARE SATISFIED THAT YOU HAVE MEASURED THE DIAMETER OF EACH CIRCLE CAREFULLY, RECORD THE MEASUREMENTS CAREFULLY IN TABLE B OF WORKSHEET 9.

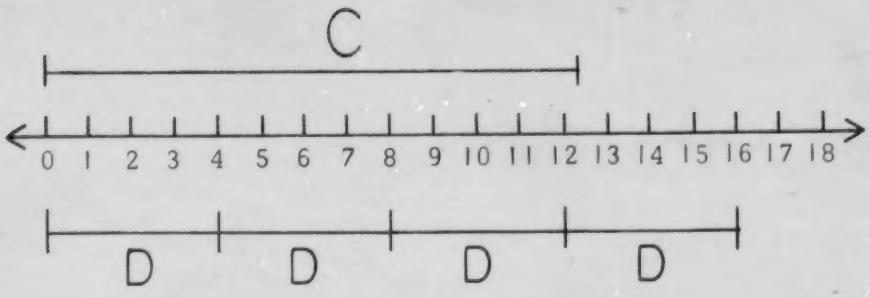
When the children have done this, ask them to discuss situations where the measurement of a diameter would be needed:

CAN YOU TELL TOR ANY REASONS WHY IT MIGHT BE USEFUL TO KNOW HOW TO MEASURE THE DIAMETER OF AN OBJECT?

Some of the answers previously given for the usefulness of circumference measurements are suitable here also, but the children may be able to think of others (e.g., fitting a rake or other tool with a new handle, finding a nut to fit a bolt, finding out whether a "round" tablecloth will cover a "round" table while the cloth is still folded in half).

Call attention to the number lines on Worksheet 9. Tell each child to choose one object from the five his group has and check to see which of his circles represent that object. Then he is to find, from his data tables, the length of the circumference of that object and plot it on number line A. On number line B, he is to plot the diameter of the circle.

Before the children work individually on number line C, demonstrate its use on a larger number line on the chalk board. Ask a child to give you the data for his circle -- its circumference and its diameter. Lay off the circumference above the number line and then lay off the diameter below the line. Starting from the end of that diameter, mark off an interval equal to another diameter. Continue marking off such intervals until one diameter interval goes beyond the end of the circumference.



Ask the class to draw the circumference of the chosen object with red crayon above the number line C. Tell the children to try to find out how many diameter-length jumps fit into the circumference by drawing the diameters below the number line C in blue crayon. Then you might ask the following:

CAN YOU TELL TOR ANYTHING YOU NOTICE ABOUT THE CIRCUMFERENCE AND THE DIAMETER OF THE CIRCLES? (There are about three diameters in a circumference.)

IF WE KNEW ONLY THE DIAMETER OF A CIRCLE, HOW COULD WE FIND ITS CIRCUMFERENCE? (We could lay off three diameters and get an idea of the circumference.)

If there is time, or if the children want to check into this further, they could make a number line on a sheet of paper, and mark off all the circumferences in Table A above the line, and all the diameters in Table B below the line. Or, using a different color of chalk for each object's measurements, you could demonstrate on a large number line on the board, all the data in the tables of Worksheet 9 as the children provide it.

Some children might like to measure cylindrical objects not used in class and report their findings. Suggested items are: kaleidoscope, piece of metal pipe, plastic or metal kitchen containers, drinking glasses, vases and jars.

SECTION 2 NUMERATION AND PLACE VALUE

PURPOSE

- To teach the use of numerals in representing the idea of number.
- To develop the child's understanding of the decimal place-value notation.
- To have the child recognize the correspondence between the real numbers and the points on a number line.
- To develop an understanding of the use of numbers in making measurements.
- To extend familiarity with the numbers up to 999.
- To give practice in addition and subtraction.

COMMENTARY

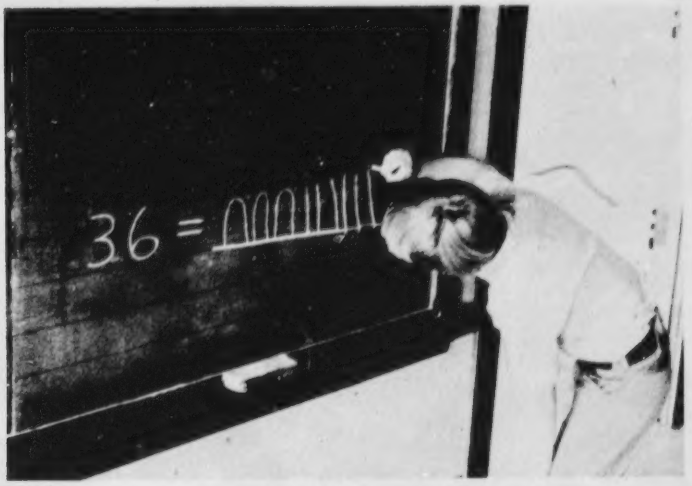
This section attempts to enlarge the child's understanding of the number concept and of the structure of the number system so that he will be able to manipulate numbers better in arithmetical operations. The child will also gain more insight into numbers by using them in making measurements.

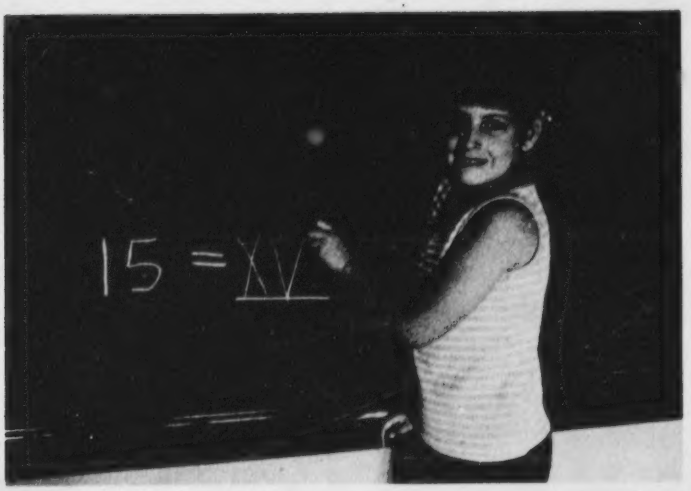
Tor can be used to good advantage in this section. The children can pretend to teach him some of the number concepts they are studying. The fact that Tor has a total of only four fingers is useful in introducing base four.

There are thirteen lessons in this section. In Lesson 5 the children study the distinction between numbers and numerals. In Lesson 6 they study T notation (using the letter T to represent 10's), and numerals to 99. Lessons 7-9 use the number line, measurement, and the addition slide rule to provide practice in addition and subtraction of two-digit numbers and to help the children understand place value. The abacus illustrates place value in another way in Lessons 10 and 11. Work with the abacus also reinforces the groundwork that has already been laid for understanding

the standard addition algorithm (rule of procedure for solving a mathematical problem). Knowledge of the numbers is extended to 999 in Lesson 12, not only to enable the children to handle larger numbers, but also to demonstrate the generality of the decimal place-value notation. Lesson 13 gives practice in counting by tens.

Lessons 14-16 are concerned with the base four system of numeration. Lessons 17 and 18 present the Egyptian and Roman systems of numeration to emphasize the superiority of the decimal place-value notation.





Lesson 5: NUMBERS VS. NUMERALS

The purpose of this lesson is to review the fact that numbers are mathematical ideas for which symbols can be written, and that these symbols are called numerals. The children are given further work with numbers to help them with their measurements and to prepare them better for their future work in mathematics and science.

MATERIALS

- pencils, counters, paper clips, etc.
- United States flag
- an apple (optional)

PROCEDURE

Activity A

Explain to the students that they are going to help teach Tor about our numeration system.

By the following procedure, demonstrate that numerals are symbols. Write on the chalkboard these ten numerals: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Tell the children that Tor wants to know what these symbols represent. Have a child read them aloud for Tor. The following discussion will bring out the idea that numerals are symbols representing numbers and that numbers are ideas describing quantities.

Have the children arrange sets of five objects, (e.g: pencils, counters, paper clips, etc.) on their desks. Ask a question of Tor, such as, "Tor, how are these sets alike?" (They all contain 5 objects. They all can be represented by the numeral 5.) Write 5 on the chalkboard. Continue by having the children arrange sets of 7 objects on their desks. Again ask the question, "How are these sets alike?" (They can all be represented by the numeral 7.) Repeat, using sets of 4 objects, 8 objects, 3 objects, etc. Tell Tor and the children that "7," "8," etc. are the names we use for representing the number of members in each set.

Tor says, "I think I understand, but I'm not quite sure. Numerals are not numbers, but they stand for numbers."

CAN YOU SHOW TOR SOMETHING IN THIS ROOM THAT IS A SYMBOL FOR OUR COUNTRY? (The United States flag.)

IS THE UNITED STATES FLAG ACTUALLY OUR COUNTRY? (No, it's a symbol for our country.)

WHEN WE SEE THE FLAG, WE ARE REMINDED OF OUR COUNTRY. THE FLAG IS A SYMBOL OF OUR COUNTRY. WE THINK OF WHAT IT STANDS FOR WHEN WE GIVE OUR PLEDGE OF ALLEGIANCE.

Now print the word "apple" on the chalkboard. Ask the children to find an apple for Tor.

IS THERE AN APPLE ON THE CHALKBOARD? (No.)

THE WORD "APPLE" ISN'T VERY TASTY, IS IT? THE WORD ON THE BOARD IS JUST A SYMBOL FOR AN APPLE.

THE SYMBOLS 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 WHICH WERE WRITTEN ON THE CHALK BOARD ARE CALLED NUMERALS. WE GIVE THEM NAMES: ZERO, ONE, TWO, THREE, FOUR, ETC. BUT THESE ARE NOT THE NUMBERS -- THEY ARE JUST THE SYMBOLS FOR THE NUMBERS.

Explain to the children that they learn many things about numbers by doing things with the symbols for numbers -- the numerals -- but numbers have properties that numerals do not. Tell them that by looking at the United States flag, they can see that our country has 50 states and 13 original colonies; but they can't know from the flag that Washington, D. C. is our capitol or that the 4th of July is Independence Day. In the same way, explain that numerals help represent numbers, but they don't tell everything about them. Say that a large part of what we call "mathematics" is the study of the properties of numbers.

Tor says, "I think I understand now."

Lesson 6: T NOTATION AND NUMERALS TO 99

This lesson reviews the numbers from one to 99 and explains how we represent them. The decimal system of place-value notation is presented as a specific example of numeration.

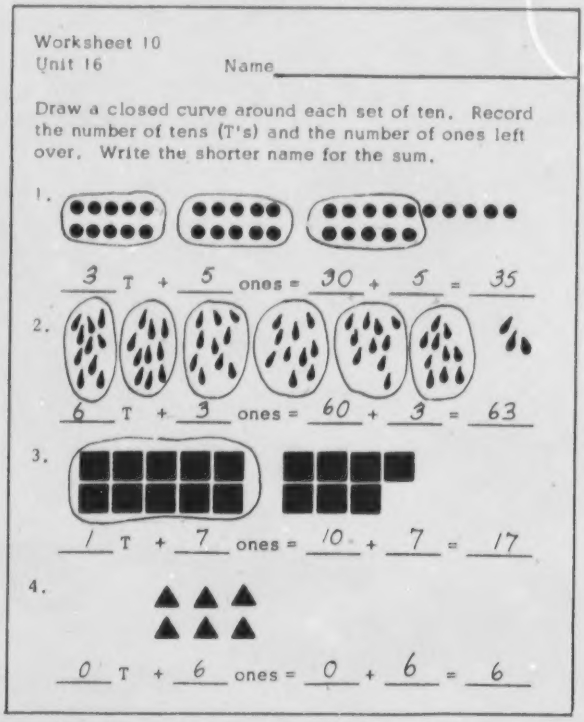
MATERIALS

- counters and flannel board
- Worksheets 10-14, colored crayons

PROCEDURE

Write several two-digit numerals on the chalk board (e.g: 16, 34, 73). Explain that Tor has asked about these numerals. What does the "34" mean? Does it mean a three and a four?

Place 34 counters on the flannel board. Have one of the children divide the counters into as many sets or groups of ten counters as he can. (3 sets of 10 counters plus 4 left over.) Have the children explain to Tor that we represent the 3 sets of 10 counters as 3 tens (or 3 x 10) or in a simpler form as 3T's. We can write a numeral for all 34 counters by using the operation of addition -- by adding the 3 tens to the 4 counters or 3T + 4. A shorter name for 3T + 4 would be 34. Therefore, when we write the numeral 34, the 3 represents the number of sets of ten, and the 4 represents the number of ones. This is called the decimal (base 10) place-value system of numerals. Repeat the above procedure with numerals such as 47, 62, 13, etc., until the children are ready to do Worksheets 10-14. Read the instructions on the worksheets to them.



Worksheet 10
Unit 16 Name_____

Draw a closed curve around each set of ten. Record the number of tens (T's) and the number of ones left over. Write the shorter name for the sum.

1. 3 T + 5 ones = 30 + 5 = 35
2. 6 T + 3 ones = 60 + 3 = 63
3. 1 T + 7 ones = 10 + 7 = 17
4. 0 T + 6 ones = 0 + 6 = 6

Worksheet 11
Unit 16 Name_____

Record the number of tens (T's) and the number of ones in the following numerals. Then write the number in the place-value chart.

16
Number of tens (T's) 1
Number of ones 6

T's	Ones
1	6

34
Number of tens (T's) 3
Number of ones 4

T's	Ones
3	4

93
Number of tens (T's) 9
Number of ones 3

T's	Ones
9	3

82
Number of tens (T's) 8
Number of ones 2

T's	Ones
8	2

42
Number of tens (T's) 4
Number of ones 2

T's	Ones
4	2

Worksheet 12
Unit 16 Name_____

Start at zero and work this way ⟶ across each row. Fill in each square with the next counting number.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Worksheet 13
Unit 16 Name_____

Use Worksheet 12 to help you fill in the boxes.

What numeral follows?

16 ?	17	8 ?	9
32 ?	33	28 ?	29
93 ?	94	47 ?	48
46 ?	47	3 ?	4
54 ?	55	74 ?	75

Worksheet 14
Unit 16 Name_____

What numeral completes the number pattern?

Color the numerals on Worksheet 12 as directed.

0, 2, 4, 6, 8, ___10___ Color these red.

9, 12, 15, 18, 21, ___24___ Color these yellow.

25, 26, 27, 28, 29, ___30___ Color these green.

38, 41, 44, 47, 50, ___53___ Color these blue.

89, 91, 93, 95, 97, ___99___ Color these brown.

Lesson 7: THE NUMBER LINE

This lesson reviews and extends the concept of the number line which is seen as still another way of representing the numbers and of learning some of their properties. The addition slide rule (which is essentially two number lines) is also used. Most of the lesson should be done as a class activity.

MATERIALS

- number line (0-100) taped to chalk board

-- for each child --

- addition slide rule
- red pen or pencil
- Worksheets 15 and 16

PROCEDURE

Draw or tape a number line on the chalk board. Suggest to the children that the number line could be used to help Tor understand more about our system of numeration. You might say, "Tor wants to know in what order we arrange our numerals." Ask such questions as:

IS THERE ANY WAY TOR CAN TELL WHICH NUMBER IS LARGER THAN ANOTHER -- BESIDES LOOKING AT THE NUMBER OF TENS AND ONES? (Yes, he can look at the number line to see which numeral is larger.)

HOW DOES THE NUMBER LINE TELL US WHICH NUMBER IS LARGER? (The numeral to the right of another numeral is always larger.)

THAT'S RIGHT. AS WE GO THIS WAY ⟶ ALONG THE NUMBER LINE, EACH NUMERAL IS ONE GREATER THAN THE PRECEDING ONE.

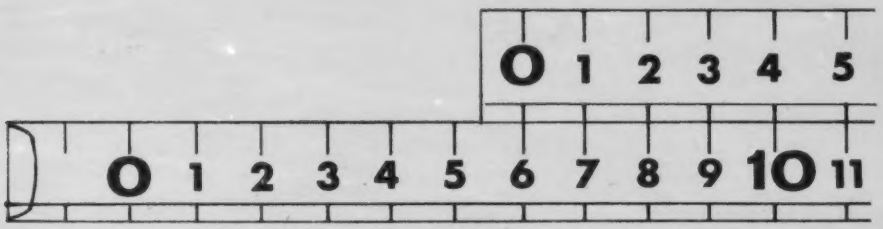
Activity A

Give each child an addition slide rule. Ask the children to describe it, eliciting that it consists of two number lines.

Ask for volunteers to demonstrate how it can be used to add two numbers. If no child remembers the procedure well enough -- from first grade -- to give a good demonstration, show the children how the two number lines of the slide rule are used.

WE ARE GOING TO ADD 3 TO 6 WITH THE ADDITION SLIDE RULE. WHERE SHOULD THE 6 BE PLACED ON THE LOWER LINE? (Straight below the zero on the upper number line.)

WHERE DO WE READ THE SUM OF 6 AND 3? (Straight down from the three of the upper line.)



Have the children use the addition slide rule and then the number line on the chalk board to show Tor how number lines are used to add numbers. Ask them to do such addition problems as 6 + 7, 33 + 6, 53 + 9, 17 + 2, etc. Continue giving addition problems until the children can use their slide rules easily.

The various rules for addition, which the children have been applying, are properties of numbers, but it is not necessary for the children to recognize this now. However, they should be able to state some of the ways (some of the rules) they have used to add numbers.

LET'S REVIEW FOR TOR SOME OF THE WAYS WE ADDED NUMBERS. (We used the decimal notation to add tens and ones; we used one number line on the chalk board; we used two number lines on the addition slide rule.)

Have the children do Worksheet 15 (as a class activity, if necessary).

Worksheet 15
Unit 16 Name _____

Use your addition slide rule to add the following.

17 + 12 = 29 4 + 5 = 9

6 + 9 = 15 84 + 9 = 93

13 + 6 = 19 5 + 8 = 13

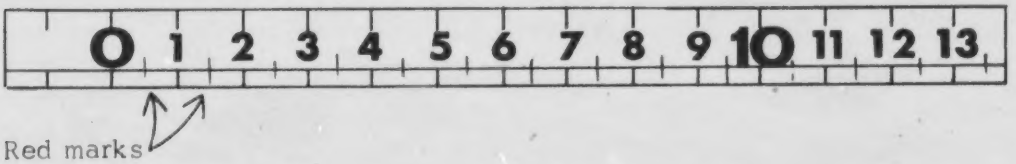
44 + 3 = 47 24 + 2 = 26

63 + 9 = 72 37 + 0 = 37

Activity B

Tell the children that Tor would like to know what we call points on the number line that are between the numerals. Have some of the children come up and find points halfway between a pair of numerals on the chalk board number line (such as $4\frac{1}{2}$, $18\frac{1}{2}$, $23\frac{1}{2}$, etc.) The number line representation helps the children see that fractional numbers exist between the whole numbers. Explain that fractional numbers must not be very different from the integers since they are both represented by points on the number line.

Give the children red marking pens or pencils. Have them mark the half-units on their addition slide rules (on the upper tape only).



Demonstrate addition problems such as these:

$5 + 2\frac{1}{2} =$ $7\frac{1}{2}$ $\quad$ $53 + 9\frac{1}{2} =$ $62\frac{1}{2}$

$17 + 3\frac{1}{2} =$ $20\frac{1}{2}$ $\quad$ $32 + 3\frac{1}{2} =$ $35\frac{1}{2}$

Continue the class work until the children seem to understand. Then have them do Worksheet 16.

Worksheet 16
Unit 16 Name_____

Use your addition slide rule (with $\frac{1}{2}$'s marked) to add the following.

$4 + 3\frac{1}{2} =$ $7\frac{1}{2}$ $\quad$ $3 + 9\frac{1}{2} =$ $12\frac{1}{2}$

$7\frac{1}{2} + 6 =$ $13\frac{1}{2}$ $\quad$ $9 + 6\frac{1}{2} =$ $15\frac{1}{2}$

$5\frac{1}{2} + 2 =$ $7\frac{1}{2}$ $\quad$ $16 + 2\frac{1}{2} =$ $18\frac{1}{2}$

$10 + 10\frac{1}{2} =$ $20\frac{1}{2}$ $\quad$ $15 + 2\frac{1}{2} =$ $17\frac{1}{2}$

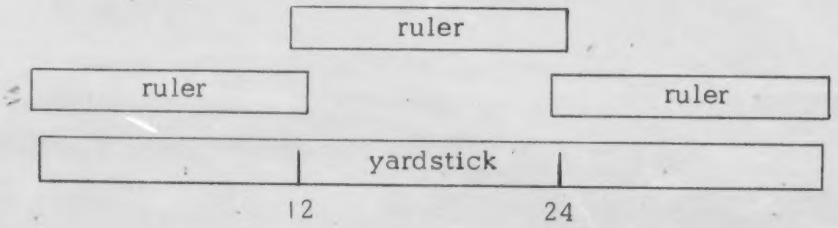
$\frac{1}{2} + \frac{1}{2} =$ 1 $\quad$ $6\frac{1}{2} + 21\frac{1}{2} =$ 28

Lesson 8: MEASURING WITH TWO-DIGIT NUMERALS

In addition to providing practice in measuring with two-digit numerals, this lesson builds the idea that numerals can be used to represent size. Concrete examples of length measurements are shown on the number line. This lesson also reinforces the concept of place value.

In Activity B the children learn the number of inches in a foot and the number of inches and feet in a yard. In order to determine the number of feet in a yard, standard 12-inch rulers are easy to use. If MINNEMAST rulers are used, allowance must be made for their extra length, either by overlapping the rulers or by arranging them in this way along the yardstick:

Method of comparing MINNEMAST rulers with a yardstick:



It is desirable for each group of children to have a yardstick and three rulers, but if this is not possible, either divide the class into larger groups or have the children take turns using the equipment. The number lines -- which the children use -- may be taped to their desks before the lesson begins.

MATERIALS

- number line (0-25) on the chalk board
- red, blue, yellow and orange chalk

-- for each group --

- 1 standard 36" yardstick
- 3 standard 12" rulers
- 1 large book

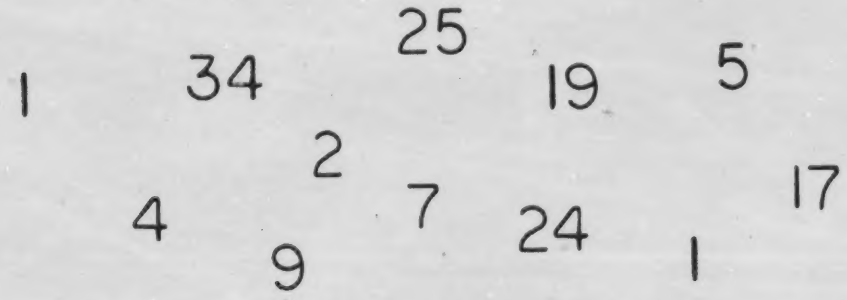
-- for each child --

- 1 desk number line (0-25)
- red, blue, yellow and orange crayons
- Worksheets 17, 18, 19 and 20

PROCEDURE

Activity A

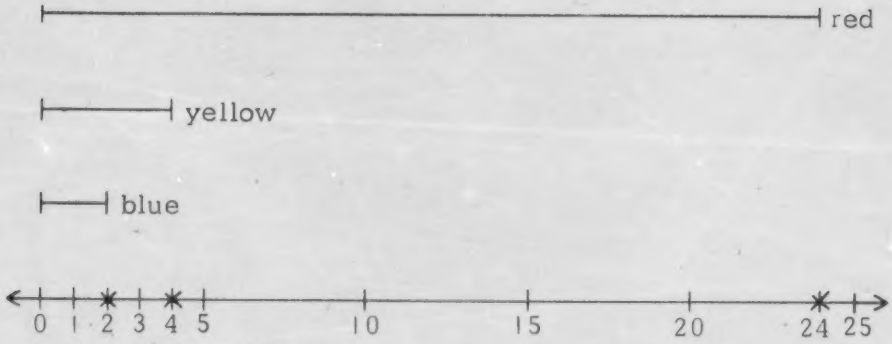
Write the following numerals on the chalk board -- in no particular order or position -- like this:



Mention that Tor is still puzzled by our numerals. He would like more information about 2's and 4's, for example. Indicating the appropriate numerals on the board, say that Tor wants to know whether 24 is the same as the sum of 2 and 4.

LET'S SEE IF WE CAN EXPLAIN TO TOR THE DIFFERENCE BETWEEN 2, 4, AND 24.

Draw a number line (0-25) on the chalk board. See that each child has a number line with the same number of units on his desk. Ask the children to locate and mark with a red crayon an X at these three points: 2, 4, and 24. Mark your chalk board number line at the same points with red chalk. Next, have each child use a ruler and a blue crayon to draw a line segment from 0 to 2; a yellow crayon to draw a line segment from 0 to 4; and a red crayon to draw a line segment from 0 to 24. Mark these on the chalk board also.



TELL WHICH LINE IS LONGEST. (The red line.)

WHICH LINE IS THE SHORTEST? (The blue line.)

Explain that the same two digits (2 and 4) were used in all the parts of the example. Bring out the fact that the length represented by combining and writing 2 and 4 as 24 is significantly longer than a length of either 2 or 4. Then, above the red line on the chalk board, represent the sum of 2 and 4 with an orange line segment from 0 to 6.

IS THE LENGTH OF 2 + 4 THE SAME AS THE LENGTH OF 24? (No.)

WHAT DOES 24 MEAN?

Answer this by writing 20 + 4 on the chalk board.

WOULD THIS SUM, 20 + 4, BE A WAY OF REPRESENTING 24? (Yes.)

CAN YOU THINK OF ANOTHER WAY TO WRITE 24? (2T + 4.)

Now tell the children that they are to think of the number line as having units which will represent inches: the numeral 24 will represent 24 inches, the 2 will be 2 inches, the 4 will be 4 inches, and so on.

On the chalk board write:

20 inches + 4 inches (or) 20" + 4"

If you use the shorter symbol (") for inches, make sure the children know what it is.

IS WHAT I HAVE WRITTEN A WAY OF REPRESENTING 24 INCHES? (Yes.)

TELL ME ANOTHER WAY TO WRITE 24 INCHES. [(2T + 4) inches.]

Repeat the activity with another set of numerals, such as 1, 5, and 15. Then ask the children to do Worksheets 17 and 18. Give as much help as necessary.

Worksheet 17
Unit 16 Name_____

Complete the following equations:

24 inches = (2 T + 4) inches

17 inches = (1 T + 7) inches

21 inches = (2 T + 1) inches

16 inches = (1 T + 6) inches

9 inches = (0 T + 9) inches

11 inches = (1 T + 1) inches

Measure the following:
(Many possible answers)

Length of Desk ≐ ___ inches = (___ T + ___) inches

Width of Desk ≐ ___ inches = (___ T + ___) inches

Worksheet 18
Unit 16 Name_____

Complete the following equations:

18 inches = (1 T + 8) inches

20 inches = (2 T + 0) inches

7 inches = (0 T + 7) inches

12 inches = (1 T + 2) inches

4 inches = (0 T + 4) inches

22 inches = (2 T + 2) inches

Measure the following:
(Many possible answers)

Length of Book ≐ ___ inches = ___ T + ___ inches

Width of Book ≐ ___ inches = ___ T + ___ inches

Activity B

Divide the class into groups and provide each group with a yardstick and three 12-inch rulers. Identify the objects for the children. Then ask them to find out (for Tor) how

many inches there are in the yardstick and how many 12-inch rulers are equal to the yardstick.

CAN YOU TELL TOR ANOTHER NAME FOR YOUR 12-INCH RULER? (It's a one-foot ruler.)

Write 1 foot and 1 ft on the chalk board.

HOW MANY FEET ARE THERE IN ONE YARD? (3.)

Give the children enough time to use the rulers in various ways to find the answers to the problems. If MINNEMAST rulers are used, see if the children can devise their own methods of taking care of the overlap when measuring the rulers along the length of the yardstick. (One method is shown in the lesson commentary.)

Ask the children to do Worksheet 19. Tell them they may work together and use their rulers and yardsticks to find the answers. The starred worksheet problems involve concepts not yet presented in the MINNEMAST materials (adding 36 and 36, which involves carrying; using the fractional notation of one-third). Use your own judgment about whether your class should try these problems with your help, whether only advanced students should do the problems, or whether the problems should be omitted.

Worksheet 19
Unit 16 Name_____

Use your rulers to help complete the following:

12 inches = 1 foot

24 inches = 2 feet

36 inches = 3 feet

36 inches = 1 yard

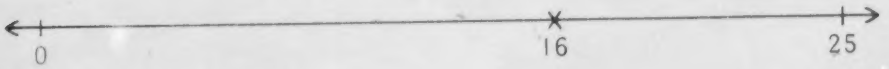
72 inches = 2 yards*

3 feet = 1 yard

1 foot = $\frac{1}{3}$ yard*

Activity C

Draw a number line (0-25) on the chalk board. Refer to the units as representing inches. Draw an X through the point representing the numeral 16.



HOW MANY FEET ARE THERE FROM 0 TO 16?

Answers to the question may vary (close to one foot, greater than one foot, etc.). Give your answer by pointing out that one foot would represent a line from 0 to 12, with 4 inches remaining. Therefore, 16 inches = 12 inches + 4 inches, or 1 foot + 4 inches. Write this on the chalk board. Repeat the activity with a 21-inch example:

21 inches = 12 inches + 9 inches, or 1 foot + 9 inches

Have the children do Worksheet 20.

Worksheet 20
Unit 16 Name_____

Use your desk number line and a ruler to help answer the following.

16 inches = 1 foot + 4 inches

21 inches = 1 foot + 9 inches

8 inches = 0 foot + 8 inches

14 inches = 1 foot + 2 inches

25 inches = 2 feet + 1 inch

12 inches = 1 foot + 0 inches

Lesson 9: ADDING TWO-DIGIT NUMERALS ON THE SLIDE RULE

The children receive more practice in using the addition slide rule to solve problems on the worksheets which make up this lesson. A few problems deal with the addition of units of weight (pounds) to show that numerals can be used to represent and add quantities other than units of length.

MATERIALS

- number line (0-100) taped to the chalk board
- addition slide rules
- Worksheets 21, 22 and 23

PROCEDURE

Let the children work the problems on the worksheets independently, if they can. If they cannot, read the instructions to them, and discuss the problems carefully so that the children will see that addition is needed to solve them.

Emphasize again the fact that addition slide rules are merely movable number lines arranged in a very convenient way for finding sums. Demonstrate by having a few children check the answers obtained on the slide rule by using the number line on the chalk board. The children should realize that when they use only one number line they have to add the second numeral to the end of the first by counting off each one; when using the slide rule (two number lines) the sum can be found much more quickly.

Worksheet 21
Unit 16 Name_____

Use your addition slide rule.

1. Billy weighs 38 pounds. Billy's boots weigh 5 pounds. How many pounds do Billy and his boots weigh together?

   38 pounds + 5 pounds = 43 pounds

2. Barbara wanted to measure a 17 inch stick with her 12 inch ruler. She found that when she placed the ruler beside the stick, the ruler was too short. How many inches short was Barbara's ruler?

   12 inches + 5 inches = 17 inches

3. Mike measured a 27 inch stick with a 12 inch ruler. He found that two lengths of the ruler were too short. How many inches short?

   12 inches + 12 inches + 3 = 27 inches

Worksheet 22
Unit 16 Name_____

Use your addition slide rule.

1. Kathy weighs 47 pounds. Kathy's winter coat weighs 8 pounds. How much will Kathy and her coat weigh together?

   47 pounds + 8 pounds = 55 pounds

2. Sammy wants to show Tor his electric train. He has two tables, one 19 inches long and one 11 inches long. If he pushes the two tables together, how long could the train track be?

   19 inches + 11 inches = 30 inches

3. Sandra has two chairs. One chair is 12 inches wide. The other is 13 inches wide. Add the widths of the two chairs.

   12 inches + 13 inches = 25 inches

Worksheet 23
Unit 16 Name_____

Use your addition slide rule.

1. Becky has two pencils. One pencil is 5 inches long. The other is 2½ inches long. Add the lengths of the two pencils.

   5 inches + 2½ inches = 7½ inches

2. Billy's book is 4½ inches long. Sue's book is 7 inches long. How many inches longer is Sue's book than Billy's?

   4½ inches + 2½ inches = 7 inches

3. Sammy is 7½ years old. Billy is 9½ years old. How much older than Sammy is Billy?

   7½ years + 2 years = 9½ years

Lesson 10: THE ABACUS

This lesson is intended as a review of the use of the abacus, but to some children this convenient device for calculating may be unfamiliar. A nine-bead (per column) abacus shows the place-value property of the decimal system of numeration. The MINNEMAST abacus has columns for thousands, hundreds, tens and ones. In this lesson the children use the abacus to find sums less than 100, so they use only the two columns representing tens and ones. All parts of the lesson may be done as class work

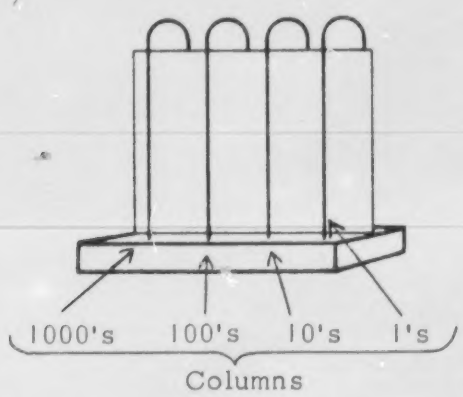
MATERIALS

- abaci (5 per class provided; 1 per child desirable)
- Worksheets 24 and 25

PROCEDURE

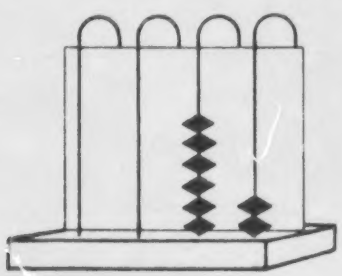
Activity A

Tell the children that Tor would like to know if there is a method of adding numerals without writing them or using a number line or slide rule. Discuss this with your class and lead one of the children to suggest using an abacus. If the children remember their work with the abacus from first grade, have one of them explain its use to Tor. If the children are not familiar with the abacus, explain its use as a calculating device. Demonstrate how the first column of beads on the right represents the number of ones and the second from the right represents the number of tens.



Show how the columns of the abacus have the same place value as the written numerals have. Give several examples, e.g:

The numeral 62 is represented on an abacus as

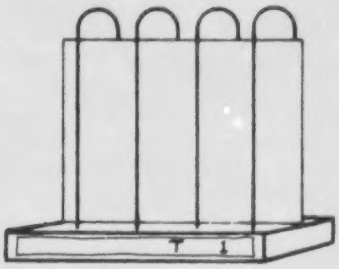


6 beads on the 10's column

2 beads on the 1's column

6 T + 2 = 62

To avoid confusion have the children label the two columns that they will be using in this lesson. Have them put tape at the base of the abacus and write 1 and T on the tape, as shown in the drawing.



Make sure the children understand that they may use no more than nine beads in any column. The 1's column may represent numerals from 0 to 9, depending on the number of beads showing. When 10 is reached, the 9 beads are slid to the back of the column and replaced by 1 bead in the 10's column. When 20 is represented, there are 2 beads in the 10's column and no beads showing in the 1's column.

Activity B

Place an abacus where all the children can see it. Illustrate a numeral such as 72 on the abacus and have one of the children tell what number is represented by the beads. Change the number of beads and have a child tell what numeral it represents. Repeat this activity until you think the children understand the process.

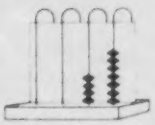
Activity C

Write a numeral on your chalk board. Have the children record it on their abaci. Check to make sure the correct number of beads is in each column. Let the children take turns writing numerals on the chalk board and representing the numerals on their abaci. Then have them complete Worksheets 24 and 25.

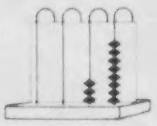
After completing the worksheets, if some children want to use their abaci to represent hundreds and thousands, permit them to experiment on their own time.

Worksheet 24
Unit 16 Name_____

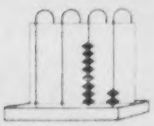
Write the numeral shown by the beads.



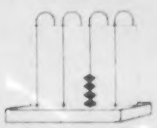
4 T's + 7 ones = 47



3 T's + 8 ones = 38



8 T's + 2 ones = 82



4 T's + 0 ones = 40

Worksheet 25
Unit 16 Name_____

Draw on each abacus as many beads as you need to show the numeral.

7	13
42	94
18	67

Lesson 11: ADDING AND SUBTRACTING WITH THE ABACUS

This lesson gives the children practice in adding and subtracting two-digit numerals with the abacus. If possible, provide each child with an abacus and prepare transparencies of the worksheets for use on an overhead projector. All of the lesson may be conducted with class participation, if you wish.

MATERIALS

- abaci (1 per child, if possible)
- Worksheets 26, 27, 28 and 29
- 1 transparency of each worksheet (optional)
- overhead projector (optional)

PROCEDURE

Activity A

Review with the children the use of the abacus. Elicit from them, or remind them, that the column at the extreme right shows the number of ones; the second column from the right shows the number of tens; etc. Emphasize again the place value of each column by representing numbers on the abacus and then in written numerals.

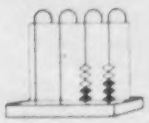
Provide each child or group of children with an abacus to use. Then project a transparency of Worksheet 26 and demonstrate the first problem. (If you have no projector, use the chalk board.) Carry out the addition on an abacus and write the answer on the chalkboard:

5 T + 5 = 55

Help the children complete this worksheet, so that they can try to do Worksheet 27 independently. Encourage them to use the abacus to find the sums.

Worksheet 26
Unit 16 Name_____

What number is shown on this abacus? 23

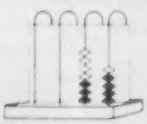


Add 32 to the abacus by drawing 3 beads on the ten's column and 2 beads on the one's column.

The new number is:

5 T + 5 = 55

What number is shown on this abacus? 34

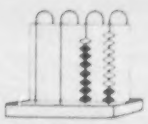


Add 41 to the abacus by drawing 4 beads on the ten's column and 1 bead on the one's column.

The new number is:

7 T + 5 = 75

What number is shown on this abacus? 72



Add 17 to the abacus.

The new number is:

8 T + 9 = 89

Worksheet 27
Unit 16 Name_____

Add. Use your abacus to find the sum.

41 + 8 = 49	16 + 2 = 18
13 + 4 = 17	4 + 13 = 17
36 + 3 = 39	3 + 36 = 39
42 + 37 = 79	21 + 43 = 64
61 + 0 = 61	37 + 30 = 67

Activity B

Demonstrate the first problem of Worksheet 28. Show the children that to subtract, they need only flip the correct number of beads to the back of the abacus. Give examples such as 34 - 21.

HOW CAN WE SUBTRACT 21 FROM 34 ON THE ABACUS? (We remove 1 bead from the ones column and 2 beads from the tens column.)

Write the answer on the chalk board in numerals:

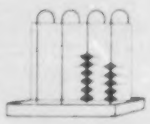
34 - 21 = 13

Have some children do subtraction problems for the class. Ask them to demonstrate both with the abacus and by writing each problem in numerals on the chalk board. Explain that on the worksheets subtraction is done by simply crossing out the specified number of beads in each column to represent flipping the beads.

Ask the children to try to complete Worksheets 28 and 29, but help them if they have difficulties.

Worksheet 28
Unit 16 Name_____

What number is shown on this abacus? 65

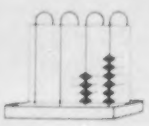


Cross out 3 beads in the one's column. Cross out 2 beads in the ten's column.

The new number is:

4 T + 2 = 42

What number is shown on this abacus? 46

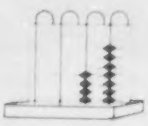


Cross out 4 beads in the one's column. Cross out 3 beads in the ten's column.

The new number is:

1 T + 2 = 12

What number is shown on this abacus? 47



Subtract 17. Cross out the correct number of beads.

The new number is:

3 T + 0 = 30

Worksheet 29
Unit 16 Name_____

Subtract. Use your abacus.

16 − 8 = 8	48 − 21 = 27
23 − 13 = 10	53 − 12 = 41
94 − 21 = 73	37 − 30 = 7
68 − 2 = 66	86 − 72 = 14
17 − 15 = 2	9 − 9 = 0

Lesson 12: THE NUMERALS THROUGH 999

The children's conception of place value is here extended to the hundreds. This gives the class some idea of the larger quantities that numerals are used to represent. After completing the lesson, the children should have an understanding of numerals through 999.

This lesson follows closely the procedure given in first grade in Lesson 39 of Unit 11, Introducing Addition and Subtraction. Lesson 39 was an optional lesson. If it was taught to your class, perhaps the children will not need to spend much time on the concepts. However, new notation for hundreds is used in this unit, e.g: since 100 means 10 tens, T·T is used to represent 100; 3T·T stands for 300. (T·T is an intermediate step towards the exponential notation, T^2.)

If you feel that your class is ready for thousands, you may present that notation also. If the class is not ready, you may wish to let some children continue to the thousands. The notation for 1000 (10 hundreds) is represented by T·T·T. In T notation, 4752 is: 4T·T·T + 7T·T + 5T + 2.

MATERIALS

-- for each child --

- 5 or more paper cups
- Worksheets 30 through 34

-- for each group of 3 or 4 --

- pile of counters (various amounts for each group, but more than 200)
- 2 or more trays or boxes large enough to hold 10 paper cups

PROCEDURE

Activity A

Divide the class into groups of three or four children. Provide each child with five paper cups, and each group with a large pile of counters and two trays. Ask the children to take counters from the group's pile and put ten in each paper

cup. When the group has ten filled cups, one child should place them in a tray. Ask the children to tell you how many counters there are in a tray filled with ten cups each holding ten counters (100). Show them how to write this in T notation as T·T. Provide enough materials so that the groups will have three-digit results for problem 1 of Worksheet 30. Each group should end up with:

- at least 2 trays of 10 cups containing 10 counters each
- cups of 10 counters insufficient to fill a tray (0-9)
- single counters left over insufficient to fill a cup (0-9)

With help from the class, demonstrate on the chalk board how the children in a group are to record the number of counters the group has:

IF A GROUP HAS 2 TRAYS FULL OF 10 CUPS, 3 CUPS OF 10'S, AND 6 COUNTERS LEFT OVER, HOW WOULD WE FILL THE BOXES IN PROBLEM 1 OF WORKSHEET 30?

Write on the chalk board:

2	3	6

Then show how the blanks in the sentence under the boxes should be filled:

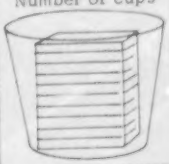
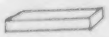
2 hundreds, 3 tens and 6 ones = 236

Have the children do problem 1 of Worksheet 30. If answers vary within a group, have the children check and correct each other.

For problem 2 of the worksheet, ask two groups to put all their trays, cups and odd counters together, and again see how many trays of ten filled cups they have, how many extra cups of tens, and how many ones are left over. Using all the counters from two groups, each child should record the results, as before.

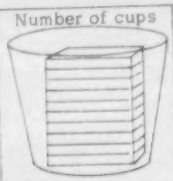
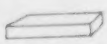
Worksheet 30
Unit 16 Name_____

1. Write in the boxes the number of counters your group has.

Number of trays	Number of cups	Number of ones
		
2	3	6

2 hundreds + 3 tens + 6 ones = 236

2. Combine your counters with those of another group. Write the number of counters in the new set.

Number of trays	Number of cups	Number of ones
		
4	2	5

4 hundreds + 2 tens + 5 ones = 425

(There are many possible answers to this worksheet.)

Next have the children complete Worksheets 31 and 32 by looking at the numerals given at the left. If any children have difficulty, remind them that T · T stands for 10 tens or 100. If you wish, have the class participate afterward in assembling the number of counters represented on the worksheets.

Conclude the activity by having the children do Worksheets 33 and 34.

Worksheet 31
Unit 16 Name_____

Fill in the blanks and boxes for each numeral.

		T·T's T's Ones
342	Number of hundreds = 3 Number of tens = 4 Number of ones = 2	3 4 2
516	Number of hundreds = 5 Number of tens = 1 Number of ones = 6	5 1 6
73	Number of hundreds = 0 Number of tens = 7 Number of ones = 3	0 7 3
407	Number of hundreds = 4 Number of tens = 0 Number of ones = 7	4 0 7

Worksheet 32
Unit 16 Name_____

Fill in the blanks and boxes for each numeral.

		T·T's T's Ones
794	Number of hundreds = 7 Number of tens = 9 Number of ones = 4	7 9 4
32	Number of hundreds = 0 Number of tens = 3 Number of ones = 2	0 3 2
347	Number of hundreds = 3 Number of tens = 4 Number of ones = 7	3 4 7
8	Number of hundreds = 0 Number of tens = 0 Number of ones = 8	0 0 8

Worksheet 33
Unit 16 Name_____

Fill in the blanks.

3 hundreds + 7 tens + 8 ones = 378

4 hundreds + 2 tens + 4 ones = 424

5 T·T + 6 T + 3 ones = 563

8 T·T + 9 T + 8 ones = 898

6 T·T + 3 T + 4 ones = 634

4 T·T + 4 T + 4 ones = 444

9 T·T + 3 T + 8 ones = 938

Worksheet 34
Unit 16 Name_____

Fill in the blanks.

549 = 5 hundreds + 4 tens + 9 ones.

210 = 2 hundreds + 1 tens + 0 ones.

832 = 8 T·T + 3 T + 2 ones.

592 = 5 T·T + 9 T + 2 ones.

821 = 8 T·T + 2 T + 1 ones.

463 = 4 T·T + 6 T + 3 ones.

617 = 6 T·T + 1 T + 7 ones.

Activity B

It is desirable for the children to know how to pronounce three-digit numerals correctly. Remind them not to use "and" between hundreds and tens:

WE SAY "FOUR HUNDRED SIXTY-TWO," NOT "FOUR HUNDRED AND SIXTY-TWO."

You may wish to write a few numerals on the chalk board and ask the children to pronounce them:

121 — One hundred twenty-one

369 — Three hundred sixty-nine, etc.

Activity C

Bring up some questions that require answers with three-digit numerals. Make up questions of your own or choose from the following:

1. How many pages are in this book?
2. How many words are on this page?
3. How many tiles are on the floor?
4. How many holes are in the pegboard?
5. How many seconds are there in two minutes?
6. How many days are in one year?
7. How much do some of the adults you know weigh?

Some of the answers to the above questions may be easy for the children to answer (questions 1, 6 and 7); others may require counting.

If you wish, you may go on to discuss questions that require answers with four-digit numerals, but this is entirely optional. Suggestions are:

1. How many seconds are there in an hour? (3600.)
2. How many miles wide is the United States? (This requires looking at a map and its scale, then measuring and converting the scale into miles.

Lesson 13: COUNTING BY TENS

Motivation for the work with tens is provided by Worksheet 35 which shows the outlines of some houses, lots and streets on a grid where each unit of length represents ten feet. Activity A prepares the children to use the worksheet and to count by tens the lengths of the sides and perimeters of a few houses and lots.

The lesson begins with the consideration of a chalk line ten feet long, so that the children will understand how much length is represented by each side of the small squares on the grid. Then the children learn how directions are shown on a map. A transparency of Worksheet 35 would be most helpful to the discussion, if you have the equipment to make and project one.

In Activity B, the children complete Worksheet 35. Activity C makes use of counters to provide more practice in counting. By counting in various ways (by twos, threes, fives, tens, etc.) the children are led to see that counting by tens is more rapid and convenient than the other methods.

MATERIALS

- foot ruler
- Worksheet 35
- transparency of Worksheet 35 (optional)
- abaci (optional)
- 60 to 100 counters for each group of 5 or 6 children

PROCEDURE

Activity A

Draw a line ten feet long on the chalk board at a height the children can easily reach, or draw it on the floor. Have a few children measure the line with a foot ruler to determine that its length is ten feet. Now have the children turn to Worksheet 35 and explain to them that each side of the small square at the bottom of the worksheet represents (stands for) a length of ten feet. In a discussion, elicit that the ten-foot

line on the chalk board would be very difficult to use as a length on a worksheet, and therefore, maps use shorter units to represent longer ones. Make sure that the children understand that each small section of the grid stands for a length of ten feet.

Next ask the children to look at the compass directions at the lower right of Worksheet 35. Hold a discussion about the way they are represented on a map and see whether the children can tell you where north, south, east and west are on the paper. Then have the children write "N" or "North" at the top of the worksheet, "S" or "South" at the bottom, "W" or "West" at the left and "E" or "East" at the right. Explain that these directions will be handy when you ask them to count off the number of feet in the south side of a house, etc. (If you wish to spend some time here having the children learn the actual directions, you may do so, but -- unless the children are facing north at their desks -- this might take some time. For the purposes of this lesson it is only necessary that the children recognize how directions are usually indicated on a map.)

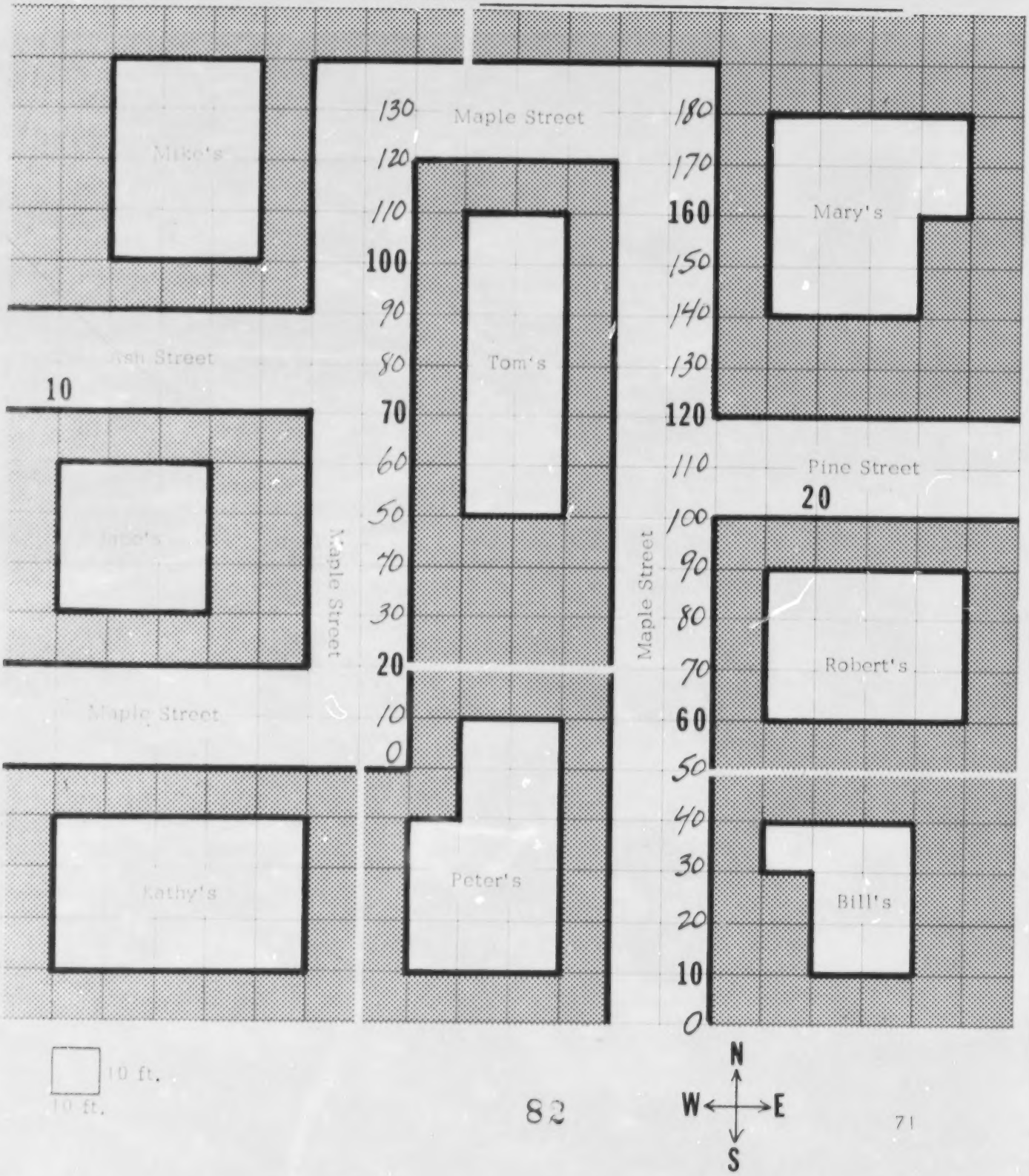
Explain that on the worksheet the regions with names printed in them will be called houses. Each house region is included in a larger region which will be called the house lot. That part of the lot which surrounds each house is shaded, so that the children can tell where the lot ends and the street or another lot begins. When two lots join, small white spaces show where each lot ends and another starts.

Have the children find Ash Street on their maps. Ask them to start at the west end of Ash Street, calling that point zero, then count by tens as they would on a number line. Remind them that in this case each mark on the line represents ten feet, so they should count along Ash Street by tens: zero, ten, twenty, thirty, forty, fifty and sixty feet.

Now ask the children to look at Jane's house, and ask how long each side of that house is. (Thirty feet.) Next ask what the perimeter -- distance around -- Jane's house is. The children may count by tens around the house to reach 120 feet for the length of the perimeter, or they may calculate 30 + 30 + 30 + 30 on their fingers, or with an abacus, or with stacks of counters each containing 10 counters.

Worksheet 35
Unit 16

Name _____



Next have the children find the length of the north side of Jane's lot (60 ft.). Then ask them to find something on the map to measure that would be the same in length as the north side of Jane's lot. (Ash Street or the south side of Mike's lot, for example.)

Continue with the following:

> WHAT IS THE MEASUREMENT OF THE EAST SIDE OF JANE'S LOT? (Fifty feet.)
>
> WHAT OTHER LENGTH IS THE SAME AS THAT OF THE EAST SIDE OF JANE'S LOT? (The north side or the south side of Kathy's house, for example.)

Ask the same kind of questions when you discuss Kathy's, Tom's and Robert's houses and lots. You need not discuss all the rectangular houses and lots if the children understand the procedure. When you think they do, direct their attention to Bill's house and lot.

> WHAT IS DIFFERENT ABOUT BILL'S HOUSE? (It is not rectangular.)
>
> HOW CAN WE MEASURE THE SOUTH SIDE OF IT?

Lead the children to measure both lines on the south side and add them. Then ask them to measure the north side.

> WHAT DO YOU NOTICE ABOUT THE NORTH AND SOUTH MEASUREMENTS OF BILL'S HOUSE? (The sum of the lengths of the two parts of the south side equals the length of the north side.)

Repeat the procedure for the east and west side of Bill's house if it is necessary to insure understanding.

Ask the children to measure Bill's lot, then have them measure Mary's and Peter's houses and lots. Guide those who need help when they try to measure Peter's, Mike's and Mary's lots.

Activity B

Before proceeding with more work on the map, ask the class to think about and discuss whether it is important to learn to make measurements and when they would need to use this skill.

Now ask the children to find the two parts of Maple Street that run in a north-south direction. Tell them to start at the south end of each part of Maple Street and write in the numerals at every point that represents ten more feet, so that they will have two separate ten-counting charts when they finish. Some numerals have been filled in to help them. Remind the children to start at zero and write that first.

When the class finishes the worksheet, correct the papers in a general discussion.

Activity C

This activity gives more practice in counting by tens. Divide your class into groups of five or six children and give each group a pile of counters (60 to 100 or so). Each child individually counts the number of counters in his group's pile, and secretly records the number on a slip of paper. (Encourage the children to think of various ways to arrange the counters -- in stacks or sets of twos, threes, fours, fives, tens, etc. The object here is to see whether the children will discover that counting by tens is more rapid and convenient.) After each child has secretly recorded his answer, he may push the counters into any arrangement he wishes for the next player. No child should tell what his count is until all have had a turn. Then, members of a group should compare and correct their answers.

Conclude the activity by asking the children to compare various ways of counting a certain number (e.g., 84) by twos, threes, fives, tens, etc. This will emphasize the method that is quickest and easiest (tens), if the children haven't already discovered it. For example:

84 counters

5, 10, 15, 20, 25, 30, 35, 40...80, 81, 82, 83, 84

vs.

10, 20, 30, 40...80, 81, 82, 83, 84

Lesson 14: TOR'S NUMBER SYSTEM -- BASE FOUR

This lesson is intended only to help the children achieve a better understanding of place value. It is not intended that they should master the concepts and ideas of the base-four numeration system. The purpose of having the children work with the base-four system, in which they group by fours, is to have them comprehend the place-value concept more easily as it applies to the decimal system, in which they group by tens.

MATERIALS

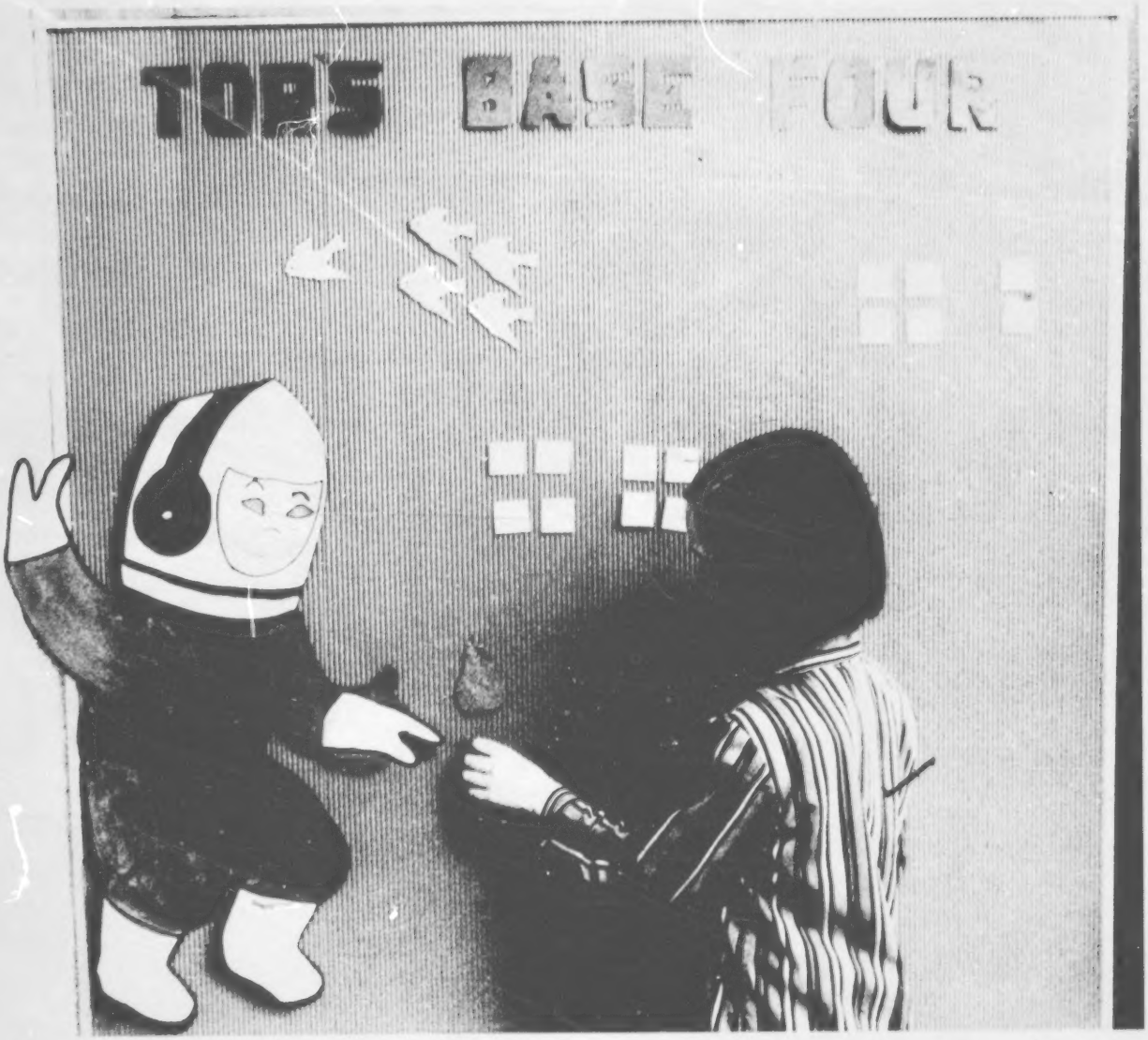
- picture of Tor on display
- flannel board
- counters for flannel board
- Minnebars
- Worksheets 36 and 37

PROCEDURE

Activity A

Ask the children to look at the picture of Tor. Ask whether the children think that the reason we count by tens is that we have ten fingers. Then see if the children can suggest what kind of a base for counting (how many numerals) Tor and his friends on Titan might use.

Say that Tor noticed that we write a T for ten, that we have ten fingers on our hands, and that we call our number system a base-ten numeration system, whereas on Titan, he and his friends each have a total of only four fingers (2 thumbs + 2 fingers = 4 fingers), so they use a base-four system. Say that Tor thinks his new friends on Earth would understand his number system best if they tried to use it a little, and he has suggested some ways for them to do so.



The photograph shows a child arranging objects on a bulletin board. Her sets represent the place values used in base 4.

Place 13 counters on the flannel board. Have a child separate the counters into groups or sets of four. (There will be one left over.)

HOW MANY GROUPS OF FOUR ARE THERE? (3.)

HOW MANY ONES ARE LEFT OVER? (1.)

Tor says, "There are three 4's and one 1. In base four I would write this number as 31 (pronounced three-one). I would use the letter F to represent the fours place and write the expanded numeral as 3 F + 1."

Write "3 F + 1 = 31 (base four)" on the chalk board.

Explain to the children that Tor uses completely different symbols on Titan to write his base-four numerals, but that you are using the base-ten symbols to write the base-four numerals, because the children already know these numerals.

Tell the children that Tor mentioned that in our base-ten system we use the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. He wants them to think about how many numerals they should use in base four and to say what they are. (0, 1, 2, and 3.)

YES, TOR SAYS THAT SINCE HE HAS ONLY FOUR FINGERS ALTOGETHER, HE NEEDS TO USE ONLY FOUR NUMERALS IN HIS BASE-FOUR SYSTEM. THE NUMERALS ARE 0, 1, 2, AND 3.

Suggest that Tor would like the children to repeat the example, this time with 11 counters on the flannel board, and then with 5. Have the children group the 11 counters into 2 F + 3, and the 5 counters into 1 F + 1.

Show the steps and notation for each example:

11 = 2 F + 3 = 23

5 = 1 F + 1 = 11

You can also explain the base-four system to the children with this analogy:

> WHEN WE WORK WITH QUARTS OF WATER, WE COUNT 0 QUARTS, 1 QUART, 2 QUARTS, 3 QUARTS; BUT WHEN WE GET TO FOUR QUARTS, WE REMEMBER THAT 4 QUARTS EQUAL 1 GALLON. THEREFORE, INSTEAD OF SAYING 4 QUARTS, WE CAN SAY 1 GALLON.

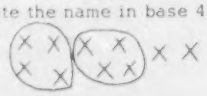
Give this example:

> 7 quarts = 4 quarts + 3 quarts = 1 gallon + 3 quarts

Conclude the activity by having the children do Worksheets 36 and 37.

Worksheet 36
Unit 16 Name_____

Draw a closed curve around each set of 4. Write the number of fours (F's) and the number of ones left over. Write the name in base 4 with numerals.

1. 

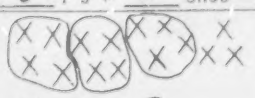
2 F's + 2 ones = 22 (base four)

2.

1 F's + 1 ones = 11 (base four)

3.

0 F's + 3 ones = 3 (base four)

4. 

3 F's + 3 ones = 33 (base four)

Worksheet 37
Unit 16 Name_____

Record the number of fours (F's) in the base-four numerals below. Record the number of ones. Then write the number in the place-value chart.

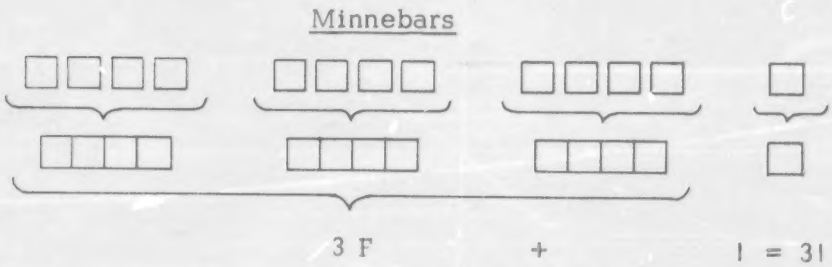
Base four		F's	ones
33	Number of fours (F's) 3 Number of ones 3	3	3
21	Number of fours (F's) 2 Number of ones 1	2	1
32	Number of fours (F's) 3 Number of ones 2	3	2
10	Number of fours (F's) 1 Number of ones 0	1	0

Activity B

Divide the class into pairs of children and give each pair nine to fifteen 1-unit Minnebars and three 4-unit Minnebars. Ask the children how many 1-unit bars are equal to one 4-unit bar (4).

Say that Tor has thought of a game the children can play with the Minnebars. Child A of each pair should start out with any number of 1-unit bars he chooses, but the number must be between nine and fifteen. Child B should start out with all three of the 4-unit bars. Now Child A should exchange as many sets of four 1-unit bars as he can for 4-unit bars. When Child A has no more sets of four 1-unit bars to exchange, he should write down the number of 4-unit bars and the number of 1-unit bars he has. That record should describe what he has in base four.

Have two children demonstrate the game. Give Child A thirteen 1-unit bars to start with and Child B three 4-unit bars. When A is through exchanging sets of fours with B, he should have three 4-unit bars and one 1-unit bar. He should write down 31. Child B should check A's base-four numeral to see that it is correct. Show the process and the notation on the chalk board:



Now have all the pairs of children play the game. Tell the class that the children should reverse their roles after they have written down the result of a game and it has been checked by the other partner. Move about the room to give assistance where needed. Remind the children to write down their results in base 4. See that the children change the number of 1-unit Minnebars for each game, but always keep it within the limit of 9 to 15 bars.

Activity C (Optional)

To show the children the number of objects represented by the numeral 21 in base ten and by the same numeral in base four, use counters, or draw on the chalk board:

Set A, 21 (base 10)	Set B, 21 (base 4)
0 0 0	0 0 0
0 0	0 0
0 0	0 0
0 0	0 0
0 0	
0 0	
0 0	
0 0	
0 0	
0 0	

WHICH SET HAS THE GREATEST NUMBER OF MEMBERS? (Set A, base 10.)

WHICH NUMERAL 21 -- BASE FOUR OR BASE TEN -- STANDS FOR THE GREATER NUMBER? (21, base 10.)

Repeat the procedure with comparisons of 13 (base 4) and 13 (base 10); 2 (base 4) and 2 (base 10); 12 (base 4) and 12 (base 10); 12 (base 4) and 6 (base 10); etc. (The last comparison will show the same number of objects -- 6 -- in each base.)

Lesson 15: TOR'S BASE-FOUR NUMBER LINE

This lesson compares base-four numeration with base-ten numeration by means of a number line. It also provides additional practice in writing base-four numerals.

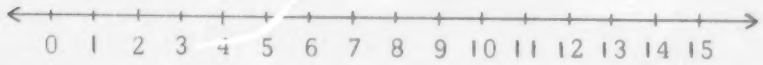
MATERIALS

- Worksheets 38 and 39

PROCEDURE

Activity A

Draw a number line on the chalk board, with the numerals 0 through 15 labeled in the familiar (base 10) manner.



Remind the children that in our decimal system of numeration, when we come to the numeral ten, we run out of basic numerals (digits) and use a combination of 1 and 0 (1 T + 0 = 10). The position or place of a digit tells its value. If it is at the extreme right, the position tells the number of ones. If it is second from the right, it tells the number of tens.

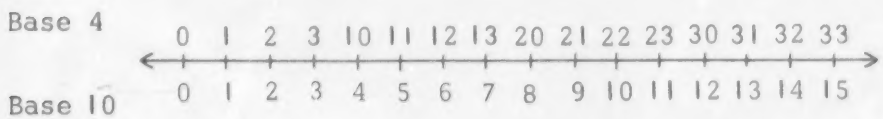
Now tell the children that they are going to help you label the number line in Tor's system, remembering that Tor can use only four digits (or basic symbols): 0, 1, 2, and 3. Label the first four corresponding points above the number line. Point out that when Tor gets to the point corresponding to our numeral for four, he has already run out of basic symbols and must use a combination of two symbols, namely 1 and 0 or 10 (1 F + 0). Label 10 above our point four. Say that the one represents the number of fours, and the zero the number of ones. Then ask:

HOW SHOULD WE LABEL THE NEXT POINT IN TOR'S BASE-FOUR SYSTEM -- THE POINT THAT CORRESPONDS TO OUR NUMERAL FOR FIVE? (1 F + 1 or 11.)

Write in 11 above the 5 on the number line. Continue this process for:

6 = 1 F + 2, or 12	11 = 2 F + 3, or 23
7 = 1 F + 3, or 13	12 = 3 F + 0, or 30
8 = 2 F + 0, or 20	13 = 3 F + 1, or 31
9 = 2 F + 1, or 21	14 = 3 F + 2, or 32
10 = 2 F + 2, or 22	15 = 3 F + 3, or 33

The chalk board number line should now show the number line in base four and base ten, like this:



Worksheets 38 and 39 provide practice in writing base-four numerals. For Worksheet 38, remind the children that they can always check (or figure out) their answers by finding out how many fours and how many ones are in the base-ten numerals given below the number line.

For Worksheet 39, remind the children that they can use Worksheet 38 for reference. The children should enjoy noticing that:

4 = 10, in base four

8 = 20, in base four

12 = 30, in base four, etc.

If the children seem to have much difficulty, use the worksheets as class activities. Put no importance at all on their memorizing or mastering the base-four system.

If your class is exceptionally bright, you might like to end the lesson by holding a discussion about other possible bases. Some children may know base two, where only zero and one

are used. Some others may like to speculate about what kind of a base Tor would have used if he had had six or eight fingers, etc.

Worksheet 38
Unit 16 Name _____

Label the number lines below with Tor's numerals. Write the base 4 numerals above the number lines. The numbers below the lines are base 10.

Worksheet 39
Unit 16 Name _____

Use Worksheet 38 to help you fill in Tor's counting numbers in the chart below. Write only numerals that are base four (Tor's base).

0	1	2	3
10	11	12	13
20	21	22	23
30	31	32	33

Lesson 16: THE BASE-FOUR ABACUS

This lesson gives the children practice in using the base-four abacus. It helps reinforce the idea of place value. The columns of MINNEMAST abaci are removable, so that many different bases can be represented on it; but for the purposes of this lesson remove all but three beads from each column.

MATERIALS

- abaci (with number of beads reduced to 3 per column)
- Worksheets 40 and 41

PROCEDURE

Activity A

Remove all but three beads from each ring (column) on the abaci. Mention to the class that Tor says this type of abacus can be used to work with his base-four number system.

CAN ANYONE TELL WHAT THE BEADS ON THE FIRST COLUMN ON THE RIGHT OF THE ABACUS STAND FOR? (The number of ones.)

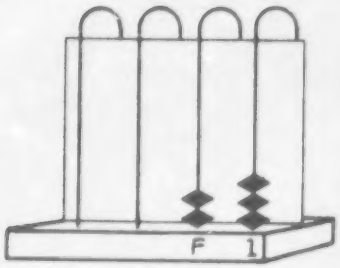
WHAT DO THE BEADS ON THE SECOND COLUMN STAND FOR? (The number of fours.)

You may also wish to ask:

WHAT DO THE BEADS ON THE THIRD COLUMN FROM THE RIGHT REPRESENT? (The number of sixteens.)

THE BEADS ON THE FOURTH COLUMN FROM THE RIGHT? (The number of sixty-fours.)

Label your abacus and have the children label theirs with a 1 at the extreme right and an F for the next column.



Show two beads on the fours column and three beads on the ones column. Write on the chalk board:

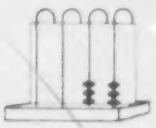
$$2\,F + 3 = 23$$
(base four)

Alter the number of beads and ask the children to write sentences using this notation to describe the base-four numeral shown on the abacus. Continue by writing several base-four numerals on the chalk board and ask the children each to record these numerals on the abacus.

When the children have had this practice, ask them to complete Worksheets 40 and 41. Keep in mind that the children are working entirely in base four here -- they are not converting back to base ten. Therefore, although 3 F + 3 (for example) might look as though the answer should be written 15, it should not, because in base four the second column refers to the number of fours, which is 3, and the first column refers to the number of ones, which in this case is also 3. If the worksheets prove too difficult for independent work, have the children do them as class activities.

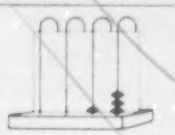
Worksheet 40
Unit 16 Name _____

Write the base four numeral shown by the beads on each abacus.



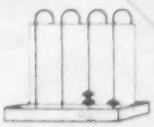
3 F + 3 ones = 33

(Base 4)



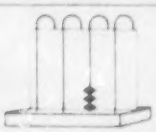
1 F + 3 ones = 13

(Base 4)



2 F + 1 ones = 21

(Base 4)



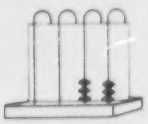
3 F + 0 ones = 30

(Base 4)

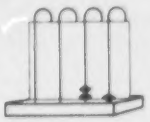
Worksheet 41
Unit 16 Name _____

Draw on the abacus the beads you need to show the base 4 numeral.

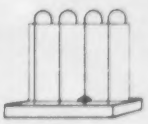
33 Base 4



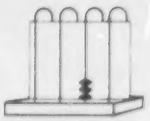
21 Base 4



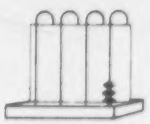
10 Base 4



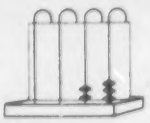
30 Base 4



3 Base 4



23 Base 4



Lesson 17: EGYPTIAN NUMERATION

This lesson is intended to help the children understand our place values in the decimal system of numeration and to make a comparison with an ancient system of numeration where the numerals had no place value. The children should not memorize the Egyptian system. The lesson should be omitted for slow learners.

The ancient Egyptian system was based on two principles -- repetition and addition. It was a decimal system (system of tens), but was primarily used for recording purposes and not for computation. The Egyptian had a set of basic symbols, of which only three are presented to the children:

a vertical stroke	|	to represent 1
a heel bone	∩	to represent 10
a scroll	9	to represent 100

Ten vertical strokes equaled one heel bone; ten heel bones equaled one scroll, etc. If the Egyptian wished to write a numeral for one item, he drew one vertical stroke (|); for two items, two vertical strokes (||); for three items, three vertical strokes (|||); and so on, up to nine strokes for nine items (|||||||||).

When the ancient Egyptian wished to write or represent ten, he used one heel bone (∩); for twenty, he used two heel bones (∩∩); for thirty, three heel bones (∩∩∩), etc. He did the same kind of repetition when writing scrolls for hundreds -- four hundred would be represented by four scrolls (9999). The four scrolls would represent 100 + 100 + 100 + 100. The Egyptian number, 99∩|, would represent 200 + 10 + 1 or 211.

You may wish to begin the lesson with a discussion of ancient Egypt. If so, pictures and films would be helpful. A list of books containing suitable pictures for display is included in the materials list for this lesson.

MATERIALS

- pictures of Egypt (optional)

  Recommended scources are:

  Life Magazine, April 1968 issues

  The River Nile by Bruce Brander, National Geographic Society, Washington, D.C., 1966

  Dictionary of Egyptian Civilization by Georges Posener, Tudor Publishing Co., New York. 1959

  Thebes of the Pharaohs by Charles F. Nims. Stein and Day. New York. 1965

  Life In the Ancient World by Bart Winer. Random House 1961, pp. 34-77

  The Golden Book of Lost Worlds by Janet Chenery. Golden Press. New York 1963, pp. 7-49

- Worksheets 42, 43 and 44

PROCEDURE

Tell your class that Tor must have spent some time at the library reading books about number systems other than ours, because he has mentioned some of the numerals used by the ancient Egyptians. If you have pictures or films about ancient Egypt, show them to the children. Pictures of the Sphinx, the Nile River, and of the wall paintings the ancient Egyptians made would be appropriate. Also show the children where Egypt is on a map or globe. Say that Tor thought the Egyptians must have been very interesting and he would like the children to hear about the Egyptian numerals.

Before starting the presentation of the Egyptian numerals, ask the children whether any of them knows what our number system is called. If no child does, provide the answer -- the Hindu-Arabic system.

Activity A

Explain to the children the meaning of the Egyptian symbols, |, ∩, and 9, i.e., that they represent one, ten, and one

hundred, respectively. Point out that the Egyptians had no symbol to represent zero. Write 17 on the chalk board. Call on a child to come to the chalk board and write the Egyptian numeral for 17. (∩|||||||.)

Help the child write the correct answer. Continue the activity by writing several examples on the chalk board and repeating the same procedure. Use numerals such as 6, 32, 94, 368, 416, etc. Have the children practice writing these numerals at their desks while the others are writing on the chalk board. Help the children complete Worksheets 42, 43, and 44.

Worksheet 42
Unit 16 Name_____

| = 1 ∩ = 10 9 = 100

Write the following in Egyptian numerals.

32 = ∩∩||

74 = ∩∩∩∩∩∩||||

142 = 9∩∩∩||

483 = 9999∩∩∩∩∩∩∩|||

774 = 9999999∩∩∩∩∩∩||||

444 = 9999∩∩∩||||

Worksheet 43
Unit 16 Name_____

1 = | 10 = ∩ 100 = 9

Write the following in our numerals.

9999999∩∩∩|||| = 734

99∩∩∩∩∩∩∩|||||| = 286

9999||| = 403

9999∩∩∩∩∩∩∩||||| = 475

99∩∩∩∩∩||||| = 255

Worksheet 44
Unit 16 Name_____

Write the sums in Egyptian, then change to our numerals.

Example:

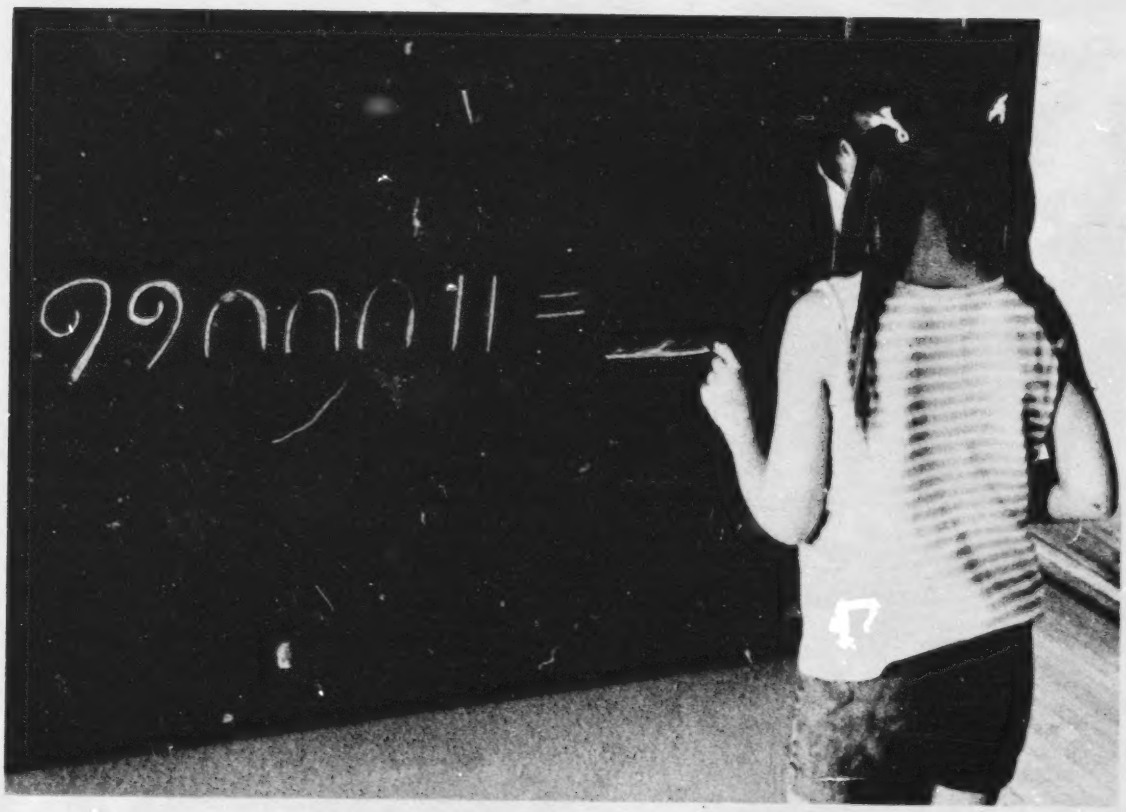
∩||| + ∩|| = ∩∩∩|||| = 45

99∩|| + 9∩| = 999∩||| = 333

9∩| + 9∩| = 99∩|| = 222

99999+ |||| = 99999|||| = 504

99+ ∩∩∩ + ||| = 99∩∩||| = 233



90

Activity B

Write on the chalk board:

462 = 𓏼𓏼𓏼𓎆𓎆𓎆𓎆𓎆||

Then ask:

HOW MANY DIGITS OR SEPARATE SYMBOLS DID WE HAVE TO USE TO WRITE 462 IN OUR SYSTEM? (3.)

HOW MANY IN THE EGYPTIAN SYSTEM? (12.)

WHICH IS EASIER TO USE? (Our system.)

Show the children that the numeral 462 may be written in any order. It can be written, for example as:

𓏼𓏼𓏼𓎆𓎆𓎆𓎆𓎆|| or 𓎆𓎆𓎆𓏼𓏼𓎆𓎆𓎆𓏼𓏼||

Explain that this is because the Egyptian system uses only the properties of addition and repetition. It follows no order and the numerals have no place value.

If you wish, repeat with other examples, such as:

962 and 𓏼𓏼𓏼𓏼𓏼𓏼𓏼𓏼𓏼𓎆𓎆𓎆𓎆𓎆||

Lesson 18: ROMAN NUMERATION

A brief investigation into the Roman numeration system should enable a child to appreciate our decimal (base ten) system more. Roman numerals are still used frequently in our society. You can see them on clock faces, in books, on cornerstones, in motion picture screen credits, in English theme outlines, and so on. Little emphasis should be placed on mastering the reading of Roman numerals -- the emphasis should be on comparing decimal numerals with Roman numerals. The children will see that the Roman system is more complex.

As with the previous lesson, you may wish to show pictures that tell the children something about the life of the ancient Romans. Three books containing suitable pictures are listed in the materials for this lesson.

MATERIALS

- pictures of ancient Rome (optional)

  Recommended sources are:

  Life in the Ancient World by Bart Winer, Random House, Inc., 1961.

  Ancient Rome by Roger Butterfield, Odyssey Press, New York, 1964.

  The Fountains of Rome by H.V. Morton, MacMillan Company, New York, 1966.

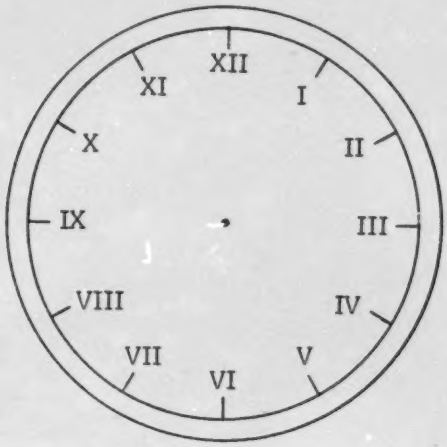
- Worksheets 45, 46 and 47

PROCEDURE

If you wish, tell the children that Tor has now been reading about the ancient Roman numeration system and would like the class to take a look at these numerals. As an introduction, discuss briefly some of the history of ancient Rome, showing pictures of Caesar, the Colosseum, etc. Have the children join in this discussion with information from films they have seen, too.

Activity A

Draw a large picture of a clock face on a piece of tagboard. Use Roman numerals to label it.



HAS ANYONE EVER SEEN A CLOCK FACE THAT LOOKS LIKE THIS? WHERE?

WHAT KIND OF NUMERALS ARE THESE? (Roman.)

WHERE ELSE HAVE YOU SEEN THESE NUMERALS? (In books, on buildings, etc.)

WHAT ARE THE DIGITS OR SYMBOLS THAT APPEAR ON THIS CLOCK FACE? (I, V, X.)

LET'S SEE IF WE CAN FIGURE OUT WHAT THESE SYMBOLS MEAN. MAYBE WE CAN COMPARE THIS CLOCK WITH OUR REGULAR CLOCK.

Draw a picture of a regular clock face and place it along side of the Roman numeral clock face.

Have the children look at Worksheet 45. Now, by one-to-one correspondence, compare the symbols as follows:

WHAT DOES THE I ON THE ROMAN CLOCK REPRESENT? (1.)

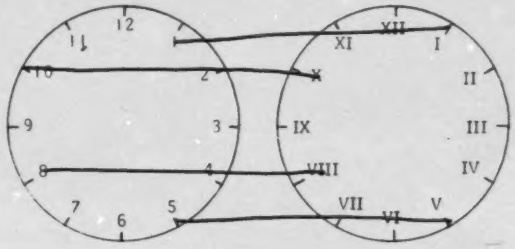
Draw a line from 1 to I. Write the I in the correct square.

WHAT DOES THE V ON THE ROMAN CLOCK REPRESENT? (5.)

Draw a line from 5 to V. Write the V in the correct square.

Worksheet 45
Unit 16

Name_____



Hindu-Arabic Roman

Complete this chart.

Hindu-Arabic	1	2	3	4	5	6
Roman	I	II	III	IV	V	VI
Hindu-Arabic	7	8	9	10	11	12
Roman	VII	VIII	IX	X	XI	XII

WHAT DOES THE X ON THE ROMAN CLOCK REPRESENT? (10.)

Draw a line from 10 to X. Write the X in the correct square.

Continue comparing all the numerals on the clock faces. Give the IV and the IX last.

CAN YOU EXPLAIN THE IV? (Four, implied as one before five.)

CAN YOU EXPLAIN THE IX? (Nine, implied as one before ten.)

Have the children complete Worksheet 45 by placing the corresponding numerals in the proper spaces. Remember to explain the notation as you proceed; e.g., XII = 10 + 1 + 1 = 12, etc.

Continue by drawing the symbols "L" and "C" on your chalk board. Explain that these symbols are Roman numerals which represent 50 and 100, respectively. Ask the children to identify orally XXXIII, LX, CXX, IX, etc.

Then have them complete Worksheets 46 and 47.

Worksheet 46
Unit 16 Name_____

Write the Roman numerals for:

1 = I	15 = XV
2 = II	20 = XX
4 = IV	50 = L
6 = VI	65 = LXV
8 = VIII	69 = LXIX
9 = IX	90 = XC
10 = X	100 = C
13 = XIII	200 = CC

Worksheet 47
Unit 16 Name_____

Write our numerals for:

VII = 7	IX = 9
XVII = 17	XIX = 19
XXII = 22	LVI = 56
IV = 4	CCXX = 220
XXIV = 24	CCC = 300

Activity B

Write 33 and XX[illegible] on the chalk board.

HOW MANY DIGITS OR SYMBOLS FOR 33 DO WE USE IN OUR SYSTEM? (2.)

HOW MANY IN THE ROMAN SYSTEM? (6.)

WHICH IS EASIER TO WRITE? (33, the Hindu-Arabic.)

DOES THE ROMAN NUMERAL HAVE PLACE VALUE? (No.)

Explain that it does have the property of order, i.e., XXXIII must be written exactly as it is. It cannot be written XXIIXI. The Roman system, as the Egyptian system, does not have a symbol for zero.

Repeat with:

L and 50
LV and 55
XXXVIII and 38
C and 100
CCXXXIII and 233

IS THE HINDU-ARABIC NUMERAL ALWAYS EASIER TO WRITE THAN THE ROMAN NUMERAL?

Not always; see L and C examples above.

SECTION 3 MEASURING WEIGHT

PURPOSE

- To develop the idea that weight is a property that can be used to describe and compare objects.
- To have the children discover that there are more precise methods of measuring the property of weight than by hefting.
- To have the children construct and use several devices for measuring weight and record the results in different ways.
- To review the idea that a fraction is a part of something, and to enlarge the children's understanding of the real number system by using fractions on the number line.
- To have the children recognize the value of standard units of measurement for the property of weight, as they did previously for the property of length.

COMMENTARY

In the seven lessons of this section the children learn more about measurement by studying the property of weight. In Lesson 19 they make and order gross comparisons of weight by hefting objects in their hands. Then the children improve their methods of measuring weight by constructing and using balances (Lessons 20 and 21). Variation in measurement is emphasized in Lesson 22 when the children develop two standard weight units to improve their measurements. In Lesson 23 fractional weight is related to the representation of numbers on the number line. In Lesson 24 the children construct another weighing device. Variation in weight measurement is demonstrated again in Lesson 25, when results obtained by different children are compared.

Lessons 22 and 25 should emphasize the fact that there is variation in all measurements. Variation is due, in part, to the inevitable errors which occur during the process of measuring. Independent measurements of the same quantity will

always differ, even though very slightly. Variation may also be due to differences in samples. Objects which are seemingly identical are never really identical and this fact also results in differing measurements of the same property. For example, one cork may look very much like another, but given sufficiently sensitive balances, one could show that each cork differs slightly in weight from all the others. Thus each object in the universe is unique, as we would find out if we could measure properties with sufficient precision.

Inaccuracy and variation are always involved in making measurements. By having the children work with weight, this section of the unit begins to illustrate these fundamental qualities of the measuring process.

Lesson 19: A NEW PROPERTY

This lesson provides experience in ordering on the basis of a property other than length. The property of weight is introduced by having groups of children order objects by hefting them in their hands to judge the relative weights. Then each child is given a set of objects to order by hefting. He records his findings for this set on Worksheet 48.

Activity A is intended to give the children an opportunity to use their observational and descriptive powers to consider a great many properties before focusing on the property of weight. It is also intended as a review of recording with symbols. A segment of the Tor story is provided below. You may find it helpful in introducing the activities.

In Activity A give the children plenty of practice in recording with symbols on the chalk board before they do the worksheet in Activity B. Show them how the members of each set are represented by lowercase letters such as a, b, c, d, and that capital W represents the weight. Thus in ordering the members of Set A by weight, a child should make this symbolic record:

Set A (a, b, c)

a -- Ping-Pong ball
b -- rubber ball
c -- clay ball

$W_a < W_b$

$W_b < W_c$

$W_a < W_c$

$W_a < W_b < W_c$

The "greater than" symbol should also be used in the comparisons. The result for Set A:

$W_c > W_b > W_a$

If you want your class to review the concept of transitivity, this lesson provides a good opportunity. Simply ask the children, "When we know that the weight of a is less than the weight of b, and that the weight of b is less than the weight of c, is it necessary to compare the weights of a and c directly?"

MATERIALS

For each group of 5 or 6 children, provide one of the Sets A-D. If you make substitutions, keep in mind that the three objects in a set should be similar in size and shape, but should vary considerably in weight.

- Set A {1 Ping-Pong ball, 1 rubber ball, 1 ball of clay}
- Set B {1 brick, 1 hard block, 1 soft block; or 1 hard block, 1 soft block, 1 closed box of similar size, etc.}
- Set C {1 irregular stone, 1 papier-maché lump, 1 irregular piece of styrofoam}
- Set D {3 covered paper cups or other containers filled with materials of three different densities} Select from such things as: soap powder, sand, sugar, salt, corks, small blocks, counters, etc.

-- for each child --

- set consisting of: {paper clip, crayon, reading book, scissors}
- Worksheet 48

Activity A

If you wish to continue the story of Tor at this point, use the following or your own version of it.

MORE ABOUT TOR

"Miss Johnson," Tor said, "I'm very happy about what I have learned about numbers and measuring with your class. I liked finding out about those ancient systems of numeration, too. And the number line, the addition slide rule, and the abacus were all very interesting. But I'm beginning to worry about two things."

"What things, Tor?" Miss Johnson asked.

"Well, the first thing is that I think I have forgotten how to use the 'greater than' and the 'less than' symbols."

"Oh, I'm sure the children will be willing to review those for you, Tor. We could start by comparing some objects for the property of length..."

"That reminds me of my second worry," Tor interrupted. "Don't Earth People ever talk about any property but length? I'm really pretty tired of measuring length. There must be many other properties to consider ..... maybe size, shape, color...?"

The children laughed. Tommy said, "I think we would all like to compare some other properties for a change." Then he picked up a Ping-Pong ball and bounced it. It bounced very high. Next he picked up a rubber ball. It bounced high, too, but not so high as the Ping-Pong ball. Finally, Tommy tried to bounce a ball of clay. It just went s-p-l-a-t and stayed on the floor.

"What property was I testing, Tor?" Tommy asked.

Tor shook his head. "I don't know," he admitted.

Sally said, "Tommy was comparing the bounciness of the three balls, weren't you, Tommy?"

"Say, that's more like it!" Tor said. "Now I'd like to hear my friends in this class describe and compare the properties of a great many objects."

"We will do better than that for you, Tor," Miss Johnson said. "The children will not only tell you how objects are alike -- they will also tell you some of the ways they are different."

"And, who knows? We may even find a very interesting new property to measure." Tommy said.

"Oh, that would be great!" Tor said.

Miss Johnson then brought out some sets of objects for the class to describe and compare. Tor was so curious about the objects that he leaned forward too far and nearly fell off his chair. Anyone could see that he could hardly wait for the lesson to start.

.....

Place enough Sets A-D around the room so that small groups of children can handle the objects and discuss the properties. Move from group to group, encouraging each group to discuss similar as well as different properties, including size, shape, color, softness, etc. (This can be either a group or a class discussion.)

Whether the children bring up the subject of weight themselves, or not, ask each group to determine how the weights of the objects compare by lifting (hefting) the objects in their hands. When each child has had an opportunity to heft at least one set of objects, have the class record the comparisons on the chalk board.

For a set of three objects, there are three binary comparisons which can be made and recorded:

$W_a < W_b$ (weight of Ping-Pong ball is less than weight of rubber ball)

$W_b < W_c$ (weight of rubber ball is less than weight of clay ball)

$W_a < W_c$ (weight of Ping-Pong ball is less than weight of clay ball)

Guide the children in recording the first set. but let them record the remaining sets on the chalk board. When this is done, ask the children to order each set:

$$W_a < W_b < W_c$$

Ask one child from each group to record the ordering of their set on the chalk board. Question the class about the relative weights of the objects shown in the ordering. Rotate groups, so that different children may check the ordering of each kind of set.

Now, if you wish to review or reinforce the concept of transitivity, ask the children whether it was necessary to compare the weight of object a with the weight of object c to determine the order of weight of three objects in a given set. From their experiences with measurements of length in first grade, some children should be able to explain that when they know that the weight of object a is less than the weight of object b, and that the weight of b is less than that of c, they do not need to compare a and c directly, because they can predict that the weight of a will be the least and the weight of c the greatest. Therefore only two binary comparisons were really needed.

Activity B

See that each child has a paper clip, a crayon, a reading book, and scissors on his desk. Ask the class to turn to Worksheet 48. Explain how the symbols are used and what the children are to do to complete this worksheet. Instruct the children to heft the objects two at a time and arrange them in order by weight on their desks -- the object of least weight at the left. Each child should record the six binary comparisons required by the worksheet. Then he should order his data on the last line.

Worksheet 48
Unit 16

Name_____

Symbol		
W_a	=	Weight of the paper clip
W_b	=	Weight of the book
W_c	=	Weight of the scissors
W_d	=	Weight of the crayon

Compare the weight of your objects by picking up one in each hand. Then write the proper symbol <, >, or ≐ in each blank.

W_a < W_c $\quad$ W_b > W_c

W_c > W_d $\quad$ W_a < W_b

W_a < W_d $\quad$ W_b > W_d

Compare the weights of all four objects. Write the letters for the objects in the blanks.

W_a < W_d < W_c < W_b

Again transitivity may be explored by asking whether it was necessary to make all six binary comparisons in order to fill in the last line of the worksheet. Some of the children may be able to tell you that sometimes only three binary comparisons will actually be needed to determine the order of weight for the four objects.

You may wish to conclude the lesson by saying that you hope Tor likes the new property -- weight -- which the children have been comparing, and that in the next lessons they will be considering some ways, other than hefting, for comparing this property.

Lesson 20: USING A SEESAW

The idea that a beam balance is useful for comparing the weights of two objects is developed in this lesson. In Activity A the children are challenged to find a method of comparing the weight of a 5-unit Minnebar with that of a paper cup by means of a ruler balanced on a pencil.

In Activity B, a seesaw is used to order all the children in a set by weight. It may not be practical for you to include all the children in the class in this set. Determine how many children you can conveniently include in the set after reading the activity. If some children are sensitive about their weight, due to obesity or extreme thinness, assign them to help you with the equipment, keep the class in order, record data, etc.

MATERIALS

-- for each child --

- pencil
- ruler
- 5-unit Minnebar
- paper cup

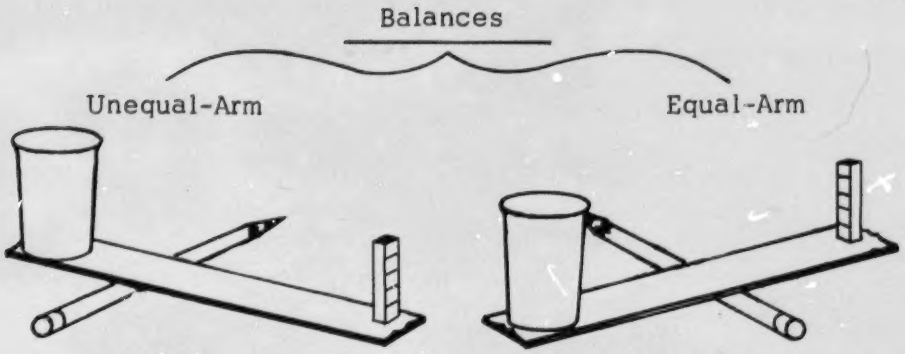
-- for the class --

- seesaw (or 6' x 8" x 2" plank balanced on a brick)

PROCEDURE

Activity A

See that each child has on his desk a pencil, ruler, 5-unit Minnebar and a paper cup. Challenge the children to discover ways to use these objects to decide whether the Minnebar is heavier or lighter than the cup. Most of the children will devise arm balances. Some may be equal-arm balances, and some may be unequal-arm balances, as shown in the diagram on the next page.



Permit students to use and observe their balances before you discuss the merits of one or the other. Invite the children to show any method of comparing objects by weight which is different from the equal-arm balance method. Then ask:

> WHAT IS THE DIFFERENCE BETWEEN AN EQUAL-ARM BALANCE AND AN UNEQUAL-ARM BALANCE? (The pencil is centered beneath the ruler for the equal-arm type.)

Have the children use their balances to answer the next two questions:

> HOW CAN YOU TELL WHICH OBJECT IS HEAVIER THAN ANOTHER IF YOU ARE COMPARING THEIR WEIGHT WITH AN EQUAL-ARM BALANCE? (The heavier object makes its end of the ruler go down.)

> HOW CAN YOU TELL WHICH OF TWO OBJECTS IS HEAVIER WHEN YOU ARE USING AN UNEQUAL-ARM BALANCE? (It is difficult, but you can tell; at balance, the shorter arm carries the greater weight.)

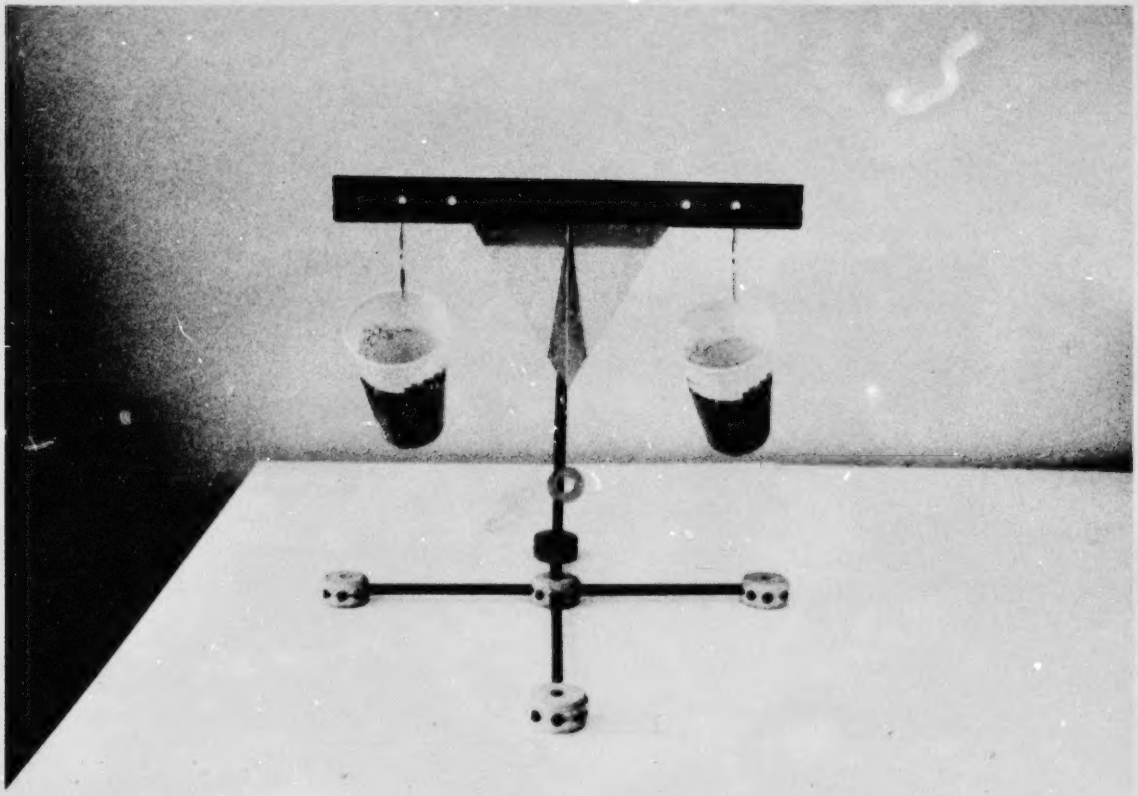
Activity B

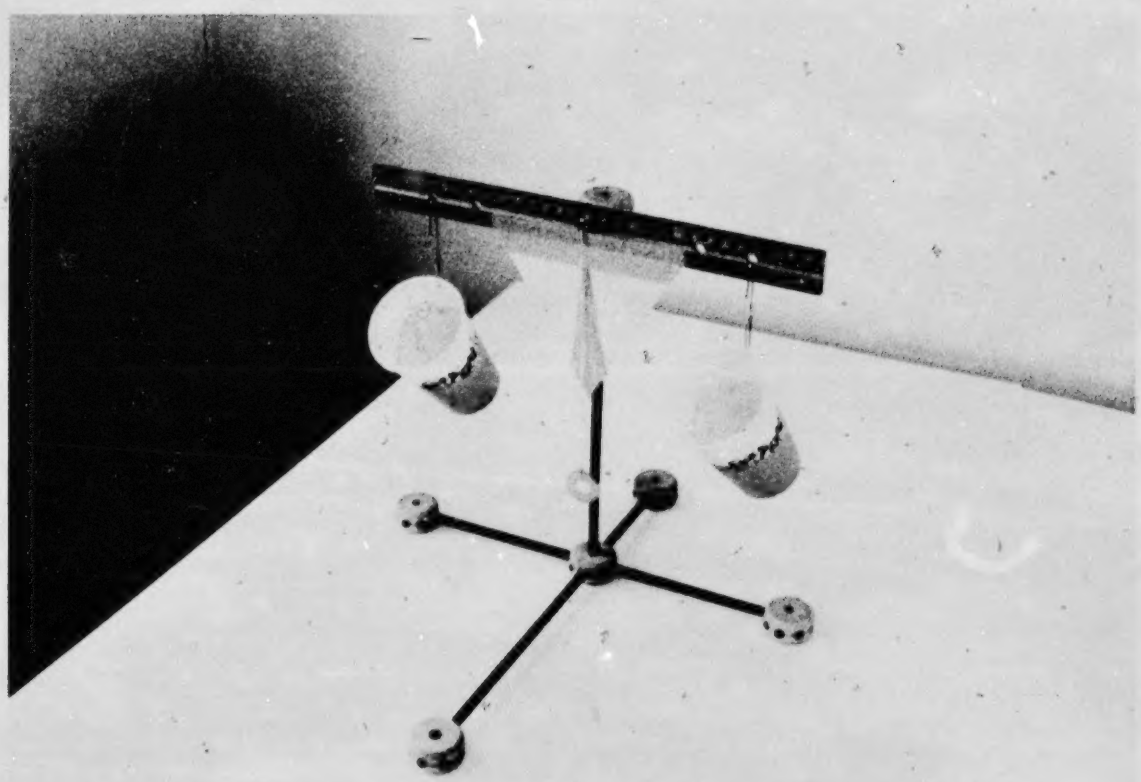
Select a set of children to be weighed. Then ask the class how you could arrange this set of children in order according to their weight. If they suggest lifting, say that this would be too difficult for you -- you want an easier method. If the idea of using an equal-arm balance does not come from the class, ask if a seesaw or teeter-totter might be useful. When this has been discussed, use a seesaw (or the plank and brick mentioned in the materials) to do the ordering.

Several seesaws in the playground could be used, if they are available and if the children will remain fairly orderly under a classmate's supervision.

As the first step in ordering the set of children by weight, select one child as the standard and sort all the children into three subsets -- the children with more weight than the standard, the children with less weight, and the children with about the same weight.

If you selected a very large set of children (or the entire class), it will be necessary to select a child in each subset as the standard for that group, and sort the members of the subset in the same way, until all the children have been arranged in order by weight.





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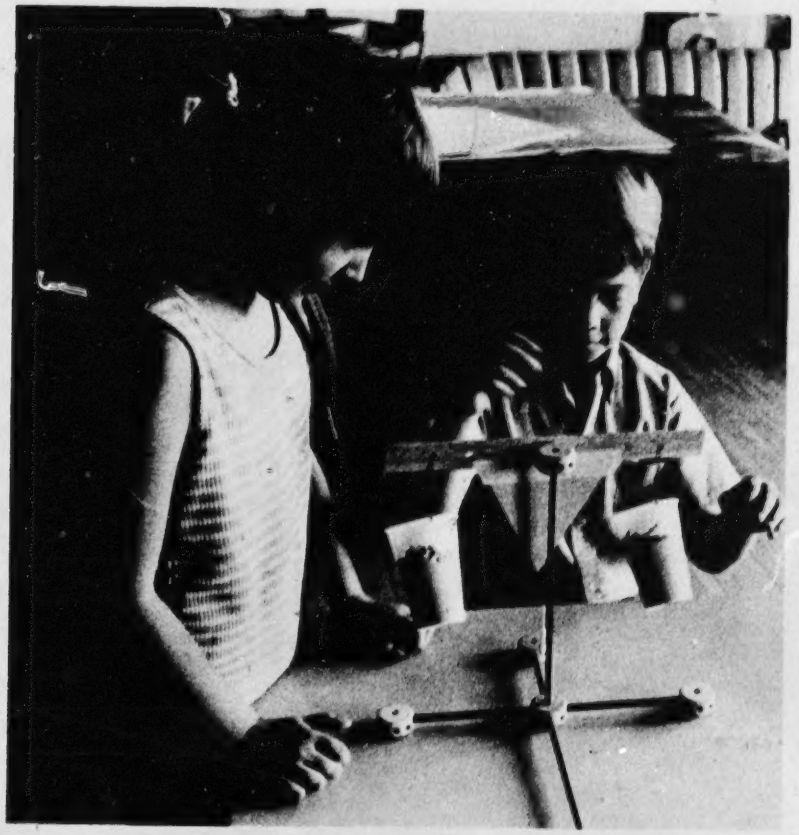
Lesson 21: BUILDING A BEAM BALANCE

This lesson familiarizes the children with the construction and use of a simple beam balance, and makes them realize the need for a standard unit of weight. After using this beam balance, the children see the advantages it has over the seesaw they used in Lesson 20.

Small groups of children construct the balances, then use them to compare the weight of two objects or of combinations of objects. It is essential that you construct a beam balance before class, using the instructions and photographs given in the lesson. This will help you select the tasks where the children need the most help, and will provide a model for them to copy. Allow one full class period for the children to assemble their balances.



MATERIALS

- 1 assembled beam balance

-- for each group --

- 1 balance kit containing: 1 ruler, 2 paper cups, 1 washer, 12" piece of thread, cardboard triangle, 2 paper clips, 10" strip of masking tape, picture hook, 2 red slotted (5") Tinkertoy rods, 1 blue slotted ($3\frac{3}{8}$") rod, 1 green slotted ($7\frac{1}{4}$") rod, 1 purple slotted ($10\frac{13}{16}$") rod, 6 round Tinkertoy connectors
- red rubber balls
- Ping-Pong balls
- pencils
- scissors
- paper cups
- corks
- small paper clips, $1\frac{1}{4}$" long

PROCEDURE

Activity A

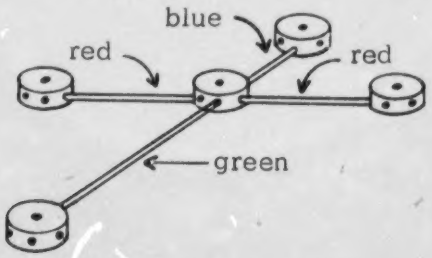
Place an assembled equal-arm balance on your desk, and drop a couple of weights into the cups. Let the children take a minute or two to look at it, and tell them that they are going to construct this balance and experiment with it.

Divide the class into groups of three or four and give one balance kit to each group. Have the children assemble their balances while you give instructions step by step. Draw diagrams on the chalk board and give help as needed.

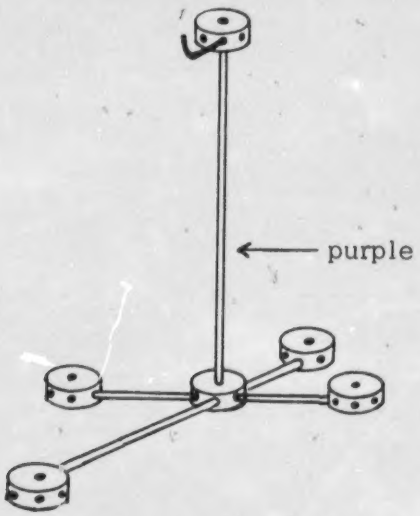
Step I. Assembling the Tinkertoy Stand

1. Screw a picture hook into one of the holes of a round connector and put it aside.

2. Assemble the base of the stand, using 5 round connectors, 1 blue rod, 2 red rods, and 1 green rod. (Have the children look at the base of your model and assemble theirs in the same way.)

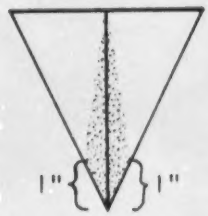


3. Insert a purple rod in the center connector of the base. Place the round connector with the picture-hook on the other end of the purple rod. Your stand assembly is now complete.

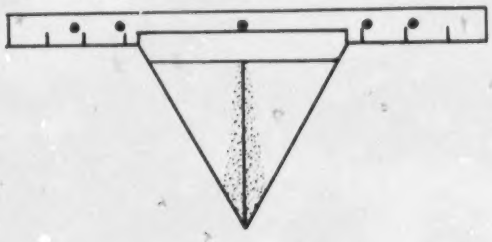


Step II. Assembling the Beam

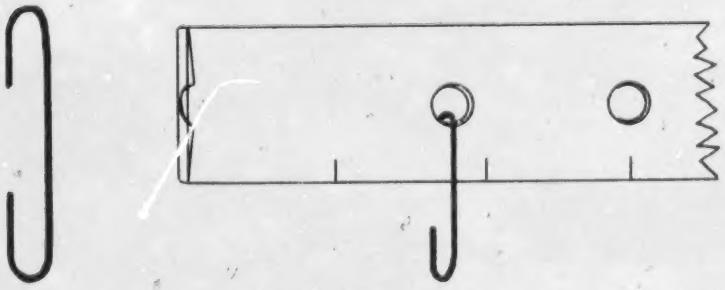
1. Take your cardboard triangle and measure up one inch on each side from the bottom. Draw a line from each of these points to the top of the center line. Color this diamond-shaped area red. (You may have to draw the lines for the children yourself and let them do the coloring.)



2. Using masking tape on both front and back sides, attach the cardboard triangle to the edge of the ruler so that the line on the triangle is centered at the hole in the middle of the ruler (i.e., line up the line on the triangle at the 6" mark).



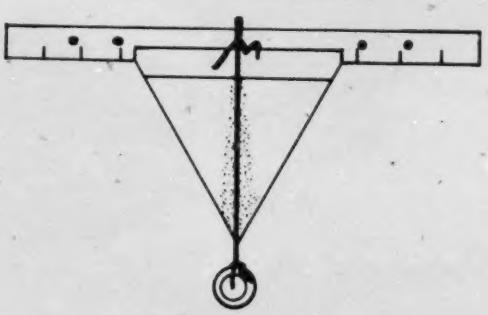
3. Unbend paper clips to make two hooks and insert one in each of the holes near the ends of the ruler.



4. Mount the balance arm on the stand by slipping the center hole of the ruler over the picture hook.

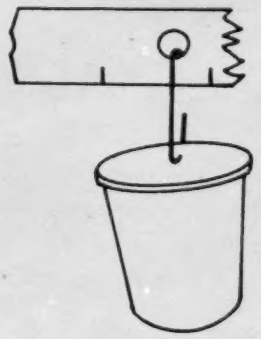
Step III. Making the Bob

1. Cut a piece of thread or thin string about 12" long.
2. Tie a washer to one end of the string.
3. Make a small loop at the other end of the string.
4. Slip the loop of the bob assembly over the picture hook.



Step IV. Attaching the Cups and Adjusting the Balance

1. Punch a small hole under the rim of each of the two cups and hang the cups on the paper-clip hooks attached to the balance arm. (Be careful to attach the cup as shown in the diagram; this lessens the chances of the children scratching their hands on the hooks when they use the balance.)



2. If your balance needs adjustment to hang evenly, take a piece of masking tape 1" long. Move it around on the balance arm until balance is obtained. Stick the tape on the back side of the balance arm at this point. (You may need to give the children considerable help with this.)

Activity B

Let the children experiment with the beam balances. Have them compare the weights of objects they compared in Lesson 20 when using the ruler balance. On the chalk board, record some of these comparisons. If possible, use the data to predict other relations of the weights of objects (transitivity). These questions will help with the discussion:

WHICH OBJECT WEIGHS LESS THAN THE PENCIL?

WHICH OBJECT WEIGHS LESS THAN THE PING-PONG BALL?

WHICH OBJECT WEIGHS THE LEAST?

After the children have had time to experiment, ask them to discuss the advantages the beam balance has over the ruler balance. The discussion should include questions that point to the following advantages of the beam balance:

1. The cups are convenient for holding objects to be weighed.

2. It is easy to decide when the weights balance.

3. It is more reliable than the ruler balance because it can be adjusted to balance evenly -- with tape, when not loaded.

4. The arms are equal in length.

5. It shows smaller differences in weight.

NOTE: The assembled beam balances are also used in Lessons 22 and 25.

Lesson 22: STANDARD WEIGHT AND VARIATION

The desirability of using a standard unit of weight is introduced in this lesson. The idea is projected by having various groups of children determine the weight of similar rubber balls by using different objects as units of weight. The children will then explore the reasons why their weight measurements vary. An attempt will be made to develop the idea of a two-unit weight standard.

MATERIALS

-- for each group --

- 1 assembled beam balance from Lesson 21
- 1 rubber ball

-- for the various groups --

- several Ping-Pong balls (Group 1)
- several pairs of scissors (Group 2)
- 4 paper cups (Group 3)
- 15 small paper clips, 1" long (Group 4)
- 5 large paper clips, 2" long (Group 5)
- 15 corks (Group 6)

-- for the class demonstration --

- 1 piece of chalk, about 1" long
- 1 small rubber eraser, about 1" long

-- for each child --

- Worksheet 49

PROCEDURE

Activity A

Divide the class into six groups of children. See that each group has an assembled beam balance, a rubber ball, and one set of the objects listed in the materials. Tell the children they are going to carry on an investigation to help the class decide on a standard unit of weight. The object to be weighed is the rubber ball.

Ask the children to weigh the ball with the beam balance, using the set of similar objects they were given as units of weight. Record on your chalk board the different combinations of weight that the different groups find. Remind the children that they are doing an investigation for the class and that they should observe the advantages and disadvantages of the units of weight they are using. Rotate your objects so that different groups will have the opportunity to weigh the ball using different objects as units of weight. After the children have had time to investigate these units, discuss the advantages and disadvantages of each weight unit, i.e., not enough room in the cup, takes too many units, never quite balances, etc. After several of these problems have been discussed, ask the children what can be done to improve their units of weight.

LET'S TAKE TWO NEW OBJECTS AND WEIGH THEM USING PAPER CUPS THE FIRST TIME, SMALL PAPER CLIPS THE SECOND TIME. ONE OF YOU WILL RECORD THE RESULTS ON THE CHALK BOARD CHART.

Objects	Number of Paper Cups	Number of Small Paper Clips
Chalk	0-1 (can't weigh)	5
Eraser	2-3	22

WHAT CAN WE LEARN FROM THIS CHART? (We use fewer cups than clips. A cup is too heavy to balance a very light object.)

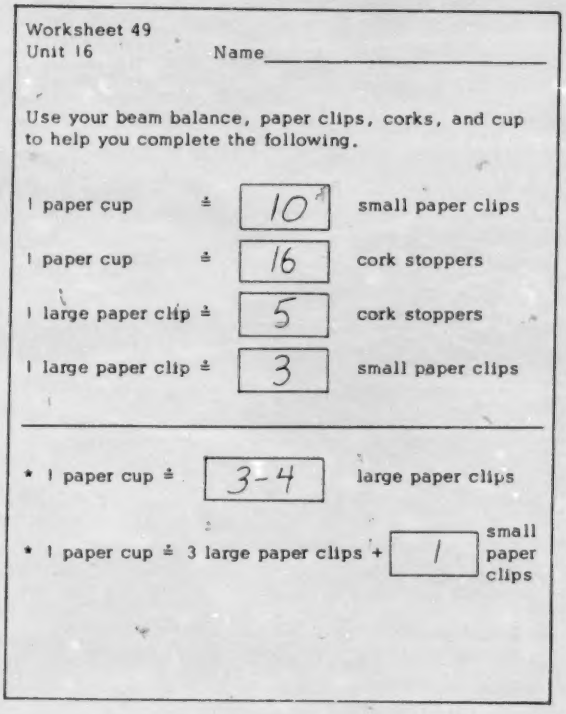
WHAT UNITS OF WEIGHT DOES THE SCHOOL NURSE USE TO WEIGH YOU? (Pounds and ounces.)

WHY DOES SHE USE TWO UNITS OF WEIGHT? (To get closer to the actual weight.)

Activity B

Divide the class into groups of three. Remind the children that they have found out that they can weigh objects with one unit but not with another. Ask each group to use beam balances to compare the weights of paper cups, large paper clips, small paper clips, and corks. Then have the children turn to Worksheet 49 and fill in the upper half. This worksheet should help them discover some facts that will eventually assist them in developing a two-unit standard of weight. Answers from each group may vary considerably.

Instruct the children that for our purposes, a reading may be taken on the beam balance when the red area is behind the white thread. Tell the children to start their investigations and record their data on the worksheet.



Worksheet 49
Unit 16 Name_____

Use your beam balance, paper clips, corks, and cup to help you complete the following.

1 paper cup ≐ [10] small paper clips

1 paper cup ≐ [16] cork stoppers

1 large paper clip ≐ [5] cork stoppers

1 large paper clip ≐ [3] small paper clips

---

* 1 paper cup ≐ [3-4] large paper clips

* 1 paper cup ≐ 3 large paper clips + [1] small paper clips

After a few minutes ask a child to read the answers on his worksheet. You may ask:

> DOES EVERYONE AGREE? (No, read more answers.)

Keep in mind that we told the children that a reading is taken within the red area. Therefore, they can obtain the weight by reading on either side of the center line. Have each group weigh one paper cup using small paper clips. For one paper cup, you might get a reading of nine small paper clips or one of ten small paper clips. After the children have finished setting up their equipment have a team of three students read and record the weight of the cup for each beam balance. Record

these weights on the chalk board. Ask the teams to explain to the class why the measurements are different. (Different sections of the red area were behind the center thread.)

Pick two teams to weigh a piece of chalk using small paper clips as the standard weight unit. The class will observe the operation so that they can answer the following questions:

HOW MANY STUDENTS OBSERVED THE SAME READING ON BOTH EQUAL-ARM BALANCES?

HOW MANY HAD DIFFERENT READINGS?

Ask each group to look at the center thread from three positions. The first at the left of the thread, the second in front of the thread and the third at the right of the thread. Spend a few minutes talking about these questions:

WHY DO READINGS VARY?

WOULD THE DIFFERENT READINGS BE CORRECT FOR OUR PURPOSES?

DO MOST MEASUREMENTS VARY?

Discuss the idea that when a measurement is made several times, we may obtain a different answer each time because of measuring error.

As a class activity, help the children complete the lower part of Worksheet 49. (The items are starred because they are difficult.) You may wish to discuss the fact that in the first example, 3 large paper clips do not weigh enough and 4 weigh too much, therefore, it is difficult to measure the weight with large paper clips. Now, 3 large paper clips plus 1 or 2 small clips is considerably closer to the weight of 1 paper cup, therefore, a better measurement.

Lesson 23: MEASURING CHILDREN'S WEIGHTS

In this lesson the children are weighed on a bath or nurse's scale. The children should assist in weighing other children, in reading the scale, and in recording the weights on the chalk board. From the data collected, the children make a histogram which graphically demonstrates the variation and distribution of their weights.

As in Lesson 20, keep in mind that some children may be sensitive about their weights. Let these children help with the tasks of weighing and recording.

MATERIALS

- bath or nurse's scale for weighing the children
- Worksheet 50
- overhead projector (optional)

PROCEDURE

Activity

Use a scale to weigh the children in your class. Ask a child to record the weights on the chalk board. This first record should be a simple one -- just a list of the weights as they are read, e.g:

43, 47, 39, 45, etc....

Have the children weigh each other, too, so that they get practice in reading the scale.

When all the weights are written on the chalk board, involve the children in a discussion of the data recorded there. Ask what the highest and lowest recorded weights are, etc. (This information will be helpful when the children make the histogram.)

After the discussion, rewrite the weight data in such a way that all similar weights are grouped together:

47, 47, 47

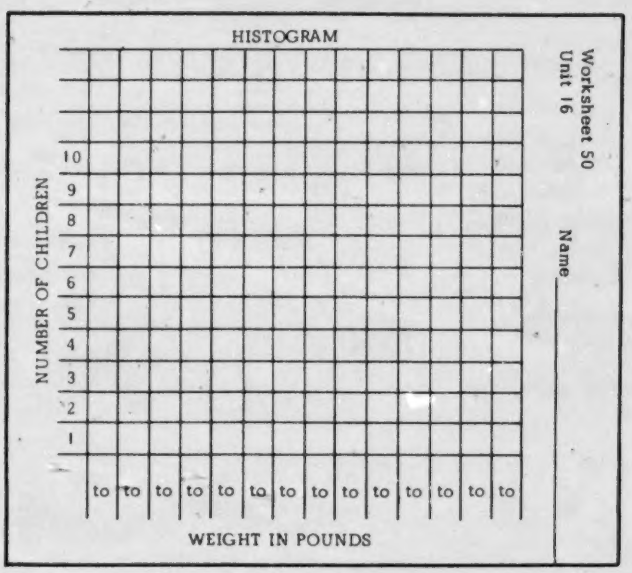
52, 52

43

45, 45, 45, etc....

Ask the children to turn to Worksheet 50 and place it on their desks so that one line is at the left and the other line is at the bottom of the paper. Tell the children to think of the bottom line as a number line where each space will represent a weight range of three pounds. Have them look at the lowest recorded weight on the chalk board and start numbering the intervals with a value that is three pounds less than that. Then they should fill in all the intervals, going to the right along the bottom. For example, if the lowest weight recorded for any child is 33 pounds, mark the first space under the number line as follows:

30
to
32



In this case the spaces below the number line would read like this:

30 to 32	33 to 35	36 to 38	39 to 41	42 to 44	45 to 47	48 to 50	51 to 53	54 to 56	57 to 59	60 to 62	63 to 65	66 to 68	69 to 71

Demonstrate the procedure on the chalk board or overhead projector. Then ask the children to listen carefully as you read the list of weights from the chalk board and to write each number, as you give it, in the first empty square above the appropriate place on the number line.

When the children have made all their entries, have them check the accuracy of their work with your own histogram, either projected or on the chalk board. Then show them how to draw heavy lines to enclose all the squares they have filled in:

HISTOGRAM

5									
4						45			
3						47			
2				39	44	45	49		
1			37	40	43	46	48	52	
	30 to 32	33 to 35	36 to 38	39 to 41	42 to 44	45 to 47	48 to 50	51 to 53	54 to 56

Discuss with the class the labeling of the horizontal and vertical axes and why it is important. (It tells the number of children in each weight group.) Ask the class to help you decide on a title for the histogram.

Have the children study their histograms and try to answer such questions as:

HOW MANY CHILDREN WEIGHED BETWEEN 39 AND 41 POUNDS?

HOW MANY MEMBERS ARE THERE IN THE SET OF CHILDREN WEIGHING 48 TO 50 POUNDS?

Try to get the class to think about the variation in weights that has been observed. Ask whether the children expected that all in the class would weigh the same. Ask about their heights and whether there is a variation in height similar to that in weight. Point out that there are no two things exactly alike in the world, and that there are always some differences among samples. Even things that look alike have very slight differences. If the lengths of two new unsharpened pencils were measured very carefully, the measurements would be slightly different.

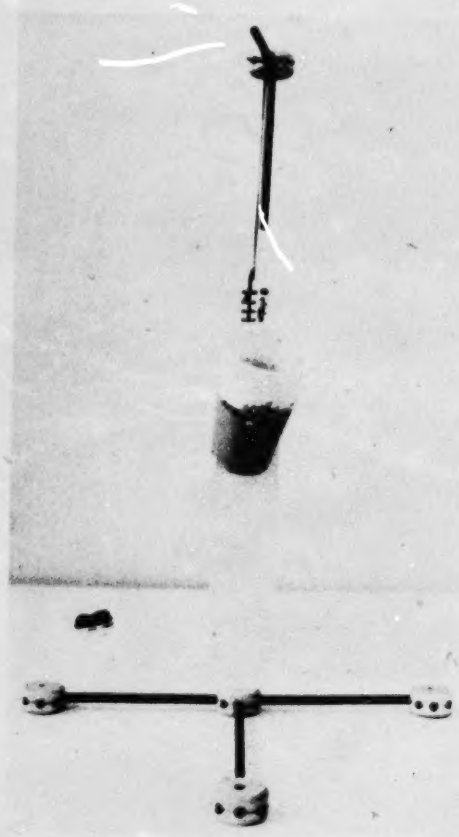
The fact that samples vary, and the impossibility of measuring things exactly (as observed in the last lesson), are the two main contributions to the children's understanding of variation.

Lesson 24: CONSTRUCTING A RUBBER-BAND SCALE

In this lesson the class develops a new weighing device, using the stand from the beam balance made in Lesson 21. Working in groups, the children build, calibrate, and use a rubber-band scale to weigh various objects. The rubber band (an elastic string) is the "spring" of the new weighing scale.

Calibrating is assigning to each point on a number line a value corresponding to a standard unit of measurement. In this lesson the units used for calibrating are paper cups and the calibrating is done vertically from top to bottom. When only one paper cup is hanging from the rubber band, the children notice where the pointer is, make a mark, and label that mark zero. As they add more cups, they make other marks and numerals until they have marked units of weight from zero to ten.

Be sure to construct a rubber band scale in advance, so that the children can look at it while building their own. Follow the four steps given in the lesson. When calibrating, notice that the marks will be increasingly farther apart, as more cups are added. This is to be expected, as the rubber string stretches more as the load increases.



MATERIALS

- Worksheet 51
- small paper clips

-- for each group --

- beam balance from Lesson 21
- a few Minnebars, other objects from desks
- 2" x 14" strip of oak tag
- large paper clip, 2" long; 1 rubber or elastic string
- 8" green Tinkertoy rod, $3\frac{1}{4}$" blue Tinkertoy rod, cylindrical Tinkertoy connector
- 11 paper cups

PROCEDURE

Activity

Ask a child to use the beam balance from Lesson 21 to weigh some small paper clips, using paper cups as his weight units. Make a record of several such measurements on the chalk board to review the comparison symbols: less than ($<$), greater than ($>$), and appears to be the same as ($\doteq$), for the class, e.g:

Weight of 1 paper cup $<$ Weight of 11 small clips $<$ Weight of 2 paper cups

or

Weight of 2 paper cups $>$ Weight of 11 small clips $>$ Weight of 1 paper cup

and

Weight of 1 paper cup $\doteq$ Weight of 10 small clips

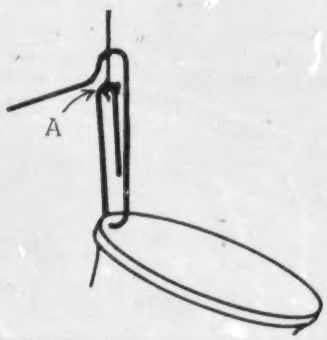
After this review of the comparison symbols, say that Tor wonders whether all weighing scales are like the beam balance just used. Ask the children to tell Tor about other weighing scales they have seen. (Some children may have noticed spring scales being used to weigh produce; others may tell about penny weighing machines; etc.) Then hold up an elastic string and ask the children whether they think this

elastic string could serve as the spring for a new kind of scale. Let the children think about and discuss this. Then divide the class into groups such that each has a beam balance from Lesson 21, and distribute the rest of the new equipment.

Use the instructions inside the four boxes to help the children make their rubber-band scales. Draw on the chalk board any diagrams that are necessary to clarify the process. Also, remind the children that they may come up and look at the rubber-band scale you have already made.

Instructions for the rubber-band scale:

Step 1. Take the large paper clip and bend one end up. Connect the clip to a paper cup by pushing the same end of the clip through the wall of the paper cup just below the rim. Tie the rubber band at point A. Tie the other end of the rubber band to the blue rod.



When the children have done this, ask each group to put a Minnebar in the paper cup. Ask what happened when they put the Minnebar in the cup. (The rubber band stretched and the cup went down.) Next ask whether this device is a good scale for weighing. Discuss and record suggestions given by the children. Give them a few minutes to experiment. Tell them to raise and lower the blue rod and see what happens. Then ask:

IS THIS EQUIPMENT YOU CAN HOLD IN YOUR HAND A GOOD WEIGHING SCALE? IS IT A POOR ONE? WHY DO YOU THINK SO? HOW CAN WE IMPROVE IT?

The children may suggest that the device they have made is easy to carry around; that it has no scale so that it is impossible to tell how many units an object weighs; that it ought to hang from a stand, etc. Elicit as many suggestions as possible; then ask whether any of this information could help in developing a new scale. After discussion, go on to the second instruction for making the rubber-band scale.

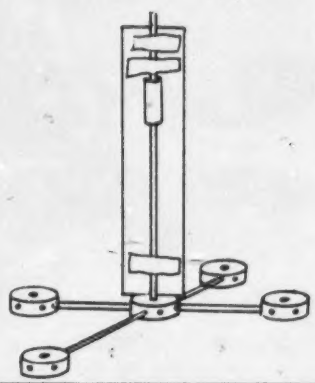
Step 2. Remove the ruler with its two cups, etc., from the purple rod of the beam balance. Place the cylindrical connector on the purple rod, then place a green rod in the top end of the cylinder. Next put the round connector on top of the green rod, so that later, when it is wanted, the balance scale may be put on the back of the stand again. Then put the blue rod in the round connector so that it is located on the front of the stand.

Have the children weigh a few objects and ask which object weighed the most. Point out that it is difficult to remember how far the rubber band stretched or how far down the cup moved. Then ask:

WHAT CAN WE DO THAT WILL MAKE IT EASIER FOR US TO REMEMBER WHAT THE DIFFERENT OBJECTS WEIGH?

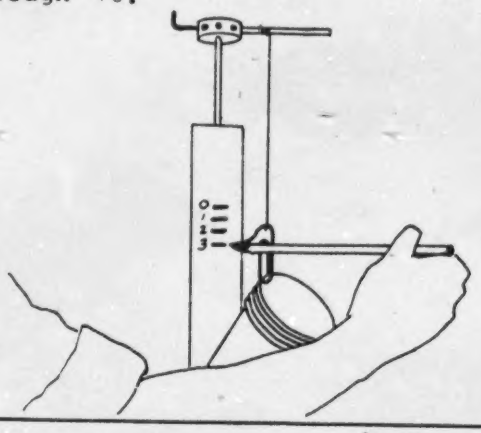
After discussion, have the children proceed to the next step of the construction. Give whatever help is needed.

Step 3. Use masking tape to fasten the oak tag strip to the upright rod. Place it so that the bottom of the oak tag rests on the round connector at the bottom.



Discuss with the children how they can make a scale on the oak tag so that objects can be weighed. Ask whether paper cups could be used as standard weight units. Finally, explain that you want the children to calibrate the rubber-band scale with paper cups as the standard units of measurement. Demonstrate what you mean by calibrating during the next step of the construction.

Step 4. Look at the turned-up end of the large paper clip. This is the pointer. Place a pencil beside the pointer and make a mark on the oak tag. Label the mark 0. Now place another paper cup inside the cup at the bottom of the rubber band. Place the pencil beside the paper-clip pointer again and mark the oak tag at this new place. Label this second mark 1. Continue adding a cup at a time to the other cups, making a new mark at each place where the pointer stops until you have marks and numerals on the oak tag from 0 through 10.



During the calibrating, some children may notice that the marks are not the same distance apart. If they cannot account for this, explain that the rubber band stretched more as the load of paper cups increased.

When the new weighing scales are ready, have the children select objects they would like to weigh and record the weights on Worksheet 51. Encourage the children to experiment with many kinds of objects to discover that not all objects can be weighed on the rubber-band scale. Ask the children to put the beam balance equipment on the back of the rubber-band scale and weigh some objects on both the beam balance and the rubber-band scale.

Worksheet 51
Unit 16 Name_____

Object	Weight
A _____	_____
B _____	_____
C _____	_____
D _____	_____
E _____	_____

Fill in the blanks with <, >, or =.

1. Weight of object A _____ Weight of object B.

2. Weight of object C _____ Weight of object D.

3. Weight of object E _____ Weight of object A.

4. Weight of object D _____ Weight of object B.

5. Weight of objects A and B _____ Weight of objects D and E.

Conclude the lesson by asking the children to discuss for Tor the advantages and disadvantages of the rubber-band scale. Children will probably respond by saying that it is convenient to be able to read the weight directly from the scale. A few may notice that the rubber band becomes stretched from continued use, so that the calibrations are not as accurate as they were at first.

Lesson 25: EXPERIMENTS WITH A BEAM BALANCE

In this lesson the children do experiments with their beam balances to find out which factors may or may not affect the weight of various objects. They select questions from Worksheets 52 and 53, or other questions of yours or of their own. All questions should require a simple experiment with the beam balance. The children work in groups, each group selecting a different question and deciding on the procedure to be used to answer the question. Each experiment involves weighing an object before and after its weight has, or has not, changed (with time, for example).

Begin the lesson with a demonstration that answers the question, "Will this birthday candle weigh more, less, or about the same, if I burn it for a while?" Have the children make predictions and tell you how to go about answering the question. Ask them to suggest a standard weight for you to use and to show you how to record the before-and-after weights, etc.

Some of the questions on the worksheets may not be practical for your class; finding substitute questions will afford the children good opportunities to devise experiments with whatever materials are available.

MATERIALS

- Worksheets 52 and 53
- birthday candle, matches

-- for each group --

- beam balance
- small paper clips, 1¼" long
- paper cups
- one selection at a time from the following materials (or substitutes): ice cubes; cotton; water; apple; sand; eggs; spring or elastic; clay; milk; pencil, chalk or crayon; bar of soap

PROCEDURE

Activity A

Suggest that Tor has been very interested in the beam balances. He thinks that the children could have fun with the balances if they used them to answer some of his many questions. Say that the first question he brought up was about a candle:

> TOR WANTS TO KNOW IF BURNING A CANDLE WILL CAUSE IT TO LOSE WEIGHT, GAIN WEIGHT, OR STAY AT THE SAME WEIGHT. HOW CAN I ANSWER HIS QUESTION?

Show a birthday candle to the class and ask what you should do to solve the problem. Help the class develop ideas for the experiment. As the following ideas are presented, carry them out for the children.

1. Weigh the birthday candle, using small paper clips as the standard weight units. Record the weight on the chalk board.

2. In a safe place, light the candle with a match.

3. Let the candle burn until it is about one inch shorter.

4. Weigh the candle again. Record the new weight.

If the children are interested, let them discuss why the candle weighed less after burning than before. (The particles changed form and passed into the air.)

Activity B

Divide the class into groups, each with a beam balance. Have each group select a problem from Worksheets 52 or 53. Instruct the children to use either paper clips or paper cups as their standard weight units. Assist the various groups in trying to develop methods for answering the questions. Show them where the equipment is that you have provided. Move from group to group, giving suggestions that will help the children get started. Let each group perform several experiments. Give as much help as is needed, but if you feel that some of the experiments will take too long or are impractical for your class, substitute others.

After a group has finished an experiment, encourage the children to discuss the results. Leave a few beam balances in a convenient place for those experiments which require evaporation over a number of days, etc.

Worksheet 52
Unit 16

Name_____

Tor wants to know:

1. Will the weight of an ice cube change after it has melted? *(Place the ice cube in a cup and weigh. Allow ice to melt and weigh again.)*
2. Do two cups of cotton weigh the same? *(Have different students fill several cups with cotton. Weigh the cups.)*
3. Will stretching a spring or rubber band change its weight? *(Weigh the spring or elastic band. Weigh a pencil or ruler. Add the weights. Stretch the spring or elastic over the pencil or ruler. Weigh.)*
4. Will the weight of a piece of clay change if we divide it into smaller pieces? *(Weigh the whole piece of clay. Divide and weigh divided pieces. Put back together again and weigh again.)*
5. Will the weight of a pencil, a crayon, or a piece of chalk change after it is used? *(Weigh a new pencil, chalk or crayon. Use and weigh again.)*
6. Does a cup of wet sand weigh more than a cup of dry sand? *(Weigh a cup of each.)*

Worksheet 53
Unit 16

Name_____

Tor wants to know:

7. Will the weight of a bar of soap change after using it? *(Weigh a new bar of soap, use and weigh again.)*
8. Will an apple lose weight sitting on the teacher's desk? *(Weigh an apple. Let it sit for 4-5 days, weigh again.)*
9. Will the weight of a cup of water change if it is allowed to sit in the classroom for a few days? Will covering the cup make any difference? *(Weigh two cups of water. Cover one and let both cups sit for 2-3 days. Weigh again.)*
10. Will the weight of a cup of water change after boiling the water? *(Weigh the water. Boil for 10 minutes. Weigh again.)*
11. Will a hard-boiled egg weigh more than a soft-boiled egg? *(Weigh the eggs. Boil one for 10 minutes, another for 2 minutes. Cool the boiled eggs and weigh each of the eggs again.)*
12. Will a carton of milk weigh the same after it turns sour? *(Weigh the carton of milk before and after the milk sours.)*

SECTION 4 NEGATIVE NUMBERS

PURPOSE

- To provide practice in directed movements along the number line.

- To introduce negative integers.

COMMENTARY

The experiences gained in this section expand the usefulness of the number line. In these two lessons direction of movement along the number line is considered in order to help explain negative numbers. The number line is shown as extending infinitely to the left as well as to the right.

Lesson 26: DIRECTIONS ON THE NUMBER LINE

This lesson gives the children practice in directed motion along a number line. It prepares for the introduction of negative numbers in the next lesson.

MATERIALS

- Worksheets 54 and 55

PROCEDURE

Activity A

Suggest to the class that Tor has noticed that many measurements -- including those of weight -- involve reading scales of numbers that are like the number line. He has asked if there are more ways of using the number line, and if there is more he should know about numbers.

Draw a number line on the chalk board. (Remember to put an arrow at each end of the number line.) Label points from 0 to 16. Then mark an X at point 8. Ask the children to tell you at what point you will be if you start at X and move 3 spaces to the right. (11.) Record this move on the board as:

Start at X and move $\overrightarrow{3}$

Explain that the arrow over the numeral shows the direction of the move.

Start at X again, but this time move 5 to the left. Record this on the board as:

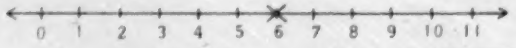
Start at X and move $\overleftarrow{5}$

Ask a child to explain the symbolism. Go through several more examples of moves to the right and moves to the left, but be sure not to end up to the left of 0.

When the children are familiar with the procedure, have them complete Worksheet 54. Remind them that they can always check their answers on the number line provided on the worksheet.

Worksheet 54
Unit 16 Name_____

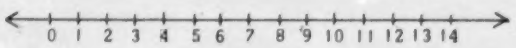
Start at X. Write a numeral in the box to show where you are after each move.



Move $\overrightarrow{4}$	10	Move $\overleftarrow{5}$	1
Move $\overleftarrow{3}$	3	Move $\overrightarrow{3}$	9
Move $\overleftarrow{0}$	6	Move $\overleftarrow{1}$	5
Move $\overrightarrow{2}$	8	Move $\overrightarrow{5}$	11
Move $\overleftarrow{6}$	0	Move $\overleftarrow{2}$	4

Worksheet 55
Unit 16 Name_____

Write a numeral in each box to show where you are after the move shown. Use the number line to help you find the answer.



$8 + \overleftarrow{4} =$	4	$10 + \overleftarrow{3} =$	7
$6 + \overrightarrow{3} =$	9	$2 + \overrightarrow{5} =$	7
$7 + \overleftarrow{5} =$	2	$1 + \overrightarrow{11} =$	12
$13 + \overleftarrow{7} =$	6	$5 + \overleftarrow{5} =$	0
$0 + \overrightarrow{11} =$	11	$6 + \overrightarrow{4} =$	10
$6 + \overleftarrow{0} =$	6	$11 + \overrightarrow{1} =$	12

Activity B

Show the children that if they did many problems starting at different points, the number lines would be full of X's. Say that they can avoid this by writing the numbers that indicate where they start on a piece of paper. For example, if they want to start at 8, they could just write 8. To show that this is followed by 3 moves to the right, they could write $8 + \overrightarrow{3}$. Demonstrate several examples on the chalk board:

$8 + \overrightarrow{4}$ (8 followed by 4 moves to the right)

$7 + \overleftarrow{6}$ (7 followed by 6 moves to the left)

$3 + \overrightarrow{2}$ (3 followed by 2 moves to the right)

Ask the children to do Worksheet 55.

Lesson 27: NEGATIVE INTEGERS

This lesson introduces negative integers to the children by extending the number line to the left of zero.

MATERIALS

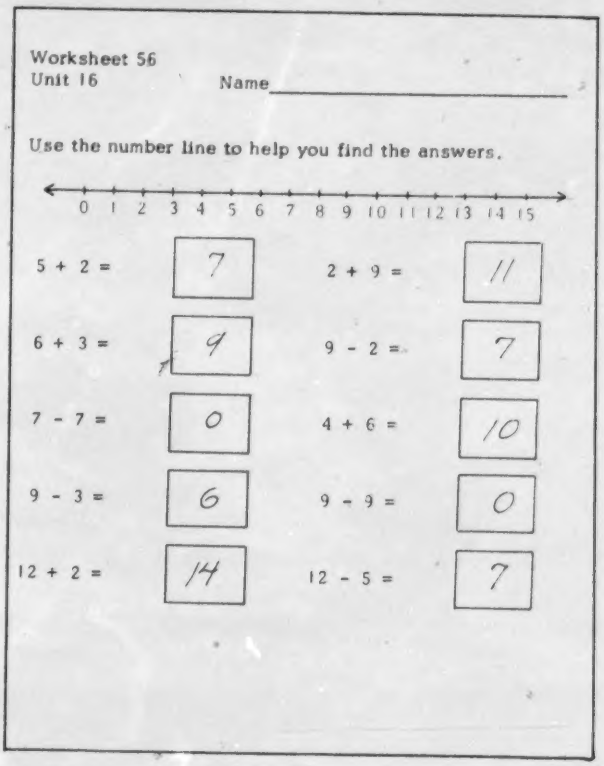
- desk number line (-15 to + 25) for each child
- Worksheets 56, 57, and 58

PROCEDURE

Activity A

Draw a number line on the chalk board. Label it from 0 to 15, leaving some space to the left of 0. Ask the children if a move of $8 + \overrightarrow{3}$ can be represented simply by $8 + 3$ and if $4 + \overrightarrow{6}$ can be written simply as $4 + 6$. (Yes.)

Now raise the question of how we could represent $6 + \overleftarrow{3}$. If no child suggests it, show the class that $6 - 3$ is the same as $6 + \overleftarrow{3}$. Repeat with $7 - 2$ for $7 + \overleftarrow{2}$, etc. Work several examples on the chalk board number line, such as $8 - 7$, $7 - 4$, $6 - 2$, $6 - 5$.


Worksheet 56
Unit 16 Name_____

Use the number line to help you find the answers.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

5 + 2 =	7	2 + 9 =	11
6 + 3 =	9	9 - 2 =	7
7 - 7 =	0	4 + 6 =	10
9 - 3 =	6	9 - 9 =	0
12 + 2 =	14	12 - 5 =	7

Now ask the children to give you another name for $6 - 3$. (3.)

Have one child go to the chalk board and go through the steps in calculating $6 - 3$. Try this with $5 - 3$, $4 - 3$, $3 - 3$. In the last case, show the class that you arrive at 0 by moving 3 steps to the left of 3.

Have the children complete Worksheet 56.

Now do 2 - 3 at the chalk board. Try to move 3 spaces to the left of 2 on the number line.

WHAT CAN WE DO IN ORDER TO SHOW THIS MOVE? (Mark units on the number line to the left of 0.)

DO WE HAVE A NAME FOR THIS LOCATION ON THE NUMBER LINE? (Not yet.)

To find the name for this point ask how many units to the left of 0 it is. (1.) Explain that since we represent movement to the left with a minus-sign, we call the point just to the left of 0, "minus one" (-1). Label the point.

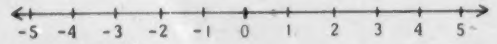
Show examples such as 2 - 4, 4 - 7, 9 - 13, and 6 - 11, to generate names for the points -2, -3, -4, and -5.

Explain that all the numbers to the left of zero are designated with a minus-sign and are called negative numbers. Ask how many negative numbers the children think there are, and whatever the number suggested, ask what the number to the left of it would be. Explain that there are just as many negative numbers as there are positive numbers (numbers to the right of zero). Do more examples with answers that are negative numbers: 7 - 9, 7 - 20, 13 - 14, 21 - 30.

Have the children complete Worksheet 57.

Worksheet 57
Unit 16 Name_____

Use the number line to help you find the answers.



4 - 6 =	-2	1 - 6 =	-5
5 - 3 =	2	4 - 2 =	2
0 - 3 =	-3	1 - 0 =	1
2 - 5 =	-3	3 - 4 =	-1
4 - 7 =	-3	5 - 7 =	-2

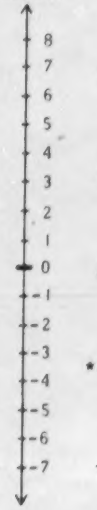
Activity B

For this activity the children should have -15 to +25 number lines on their desks. Have them rotate the number line counter-clockwise from a horizontal position to a vertical position. Be sure that the positive numerals are above 0 and the negative numerals are below 0.

Explain to the children that they now have a number line like that on a thermometer. Very cold temperatures, such as those below zero, are represented by negative numbers. For example, 14 below 0 is written as -14. Give the children practice in adding and subtracting with this vertical number line. Point out that 4 + 2 means that they must start at 4 and move upward two units (4 + 2 = 6) and 4 - 6 means they must start at 4 and move downward 6 units (4 - 6 = -2). You may wish to use problems dealing with temperature, e.g: 17 - 10 = 7, etc.

Worksheet 58
Unit 16 Name_____

Use the vertical number line to help you fill in the boxes.



7 - 2 =	5	5 + 3 =	8
3 - 6 =	-3	2 - 3 =	-1
8 - 10 =	-2	6 - 7 =	-1
*-1 - 2 =	-3	1 + 5 =	6
-1 + 5 =	4	3 - 7 =	-4

Have the children complete Worksheet 58.

Activity C: Games

1. "Take a Trip"

Draw a large number line on the floor. Mark off points about 12 inches apart and label them from -15 to +15.

Divide the children into groups of five. One group will play at a time. Label several 3" x 5" cards with numerals from -1 to +3. The cards should be shuffled. Each member of the group gets five moves. He draws five cards at random, starts at zero and steps along the number line as indicated by the numerals on his cards, in the order in which he drew them. The point where he is after the fifth move is recorded. You can decide beforehand as to what rule will determine the winner -- high score, low score, closest to zero, etc. Each time the children play, vary the rules for winning.

2. "Captain, May I?"

Play this with a number line on the floor. Have the captain say the number of units and the direction of each child's steps. Give all children a turn at being captain.

SECTION 5 TEACHING TOR TO USE OUR MONEY (Lesson 28)

This lesson introduces the relations among the different American coins. It reinforces the idea of grouping as the basis of place-value notation. Pennies, nickels, dimes, and a quarter are considered. You may wish to include work with half-dollars and dollars also, but this is optional.

When you use the parallel number lines to show relations among the coins, do not try to develop any understanding of the multiplicative relations involved. This is done in Unit 17, Introducing Multiplication and Division. Do not ask the children to make parallel number lines of their own.

Teaching Tor is a helpful device for going over material with which the children may already be familiar, so a segment of the Tor story is provided to introduce the lesson. Then, since this is the last lesson of the unit, another segment is provided at the end; it describes Tor's departure for his home.

MATERIALS

- pennies, nickels, dimes, quarters (halves and dollars optional); or facsimiles from the coin sheet
- addition slide rules
- Worksheets 59, 60 and 61

PROCEDURE

Activity A

TOR FINDS OUT ABOUT MONEY

Tommy reached into his pocket to pull out his handkerchief, and a penny fell to the floor. It made a tinkling sound, and Tor picked it up and looked at it curiously. "This seems to be a piece of copper," he said, "but it has a very interesting design. What is it?"

"It's money," Tommy said. "It's called a penny, Tor."

"What do you do with it? What is it used for?" Tor asked.

"It's for buying things."

"Buying?" Tor was puzzled. "What does 'buying' mean?"

"We go to the store and ask the man there for a piece of candy or a balloon or anything he has that we want. He gives us the candy or balloon, or whatever we want, and we give him the penny," Tommy said. "We just exchange the penny for what we want."

"If I gave the man in the store this penny, could I buy a big chair like the one you are sitting on?" Tor asked.

"Oh, no. That would cost dollars, not pennies."

"What are dollars?"

"Well, dollars are money too, but they buy more than pennies do. A dollar will buy as much as one hundred pennies could."

Miss Johnson spoke up. "Why don't we all help Tor find out how our money system works?"

So the children told Tor that people worked and were paid money, and that when they needed something, they could take the money to the store and exchange it for what they needed. The children explained that some things cost just a penny or a few pennies, but other things cost many dollars.

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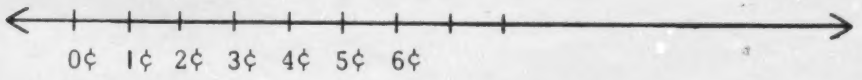
Remind the children that Tor is always interested in how Earth People write things down. Write "1 penny" on the

Unit 16, Coin Sheet

chalk board, and ask if there is another way this could be written. You should finally have on the chalk board the following:

1 penny = 1 cent = 1¢

With the children's help, develop a list on the chalk board with as many examples as are needed to establish this method of notation. Your list should include 5¢, 10¢, and 25¢.



IF WE WANT TO SOLVE A PROBLEM ABOUT MONEY, DO WE NEED THE CENTS SYMBOLS ON THE NUMBER LINE? (No.)

Erase the cents symbols you just wrote, and fill in more numerals to emphasize that this is just like any other number line, even though you are representing pennies.

Activity B

For this activity use real coins or the printed coins provided in this manual and in the Student Manuals.

TOR IS A VERY PRACTICAL FELLOW, AND HE WANTS TO KNOW IF EARTH PEOPLE GET TIRED CARRYING AROUND SO MANY PENNIES. WHAT SHALL WE TELL HIM? (We have other coins worth more than pennies.)

Have the children name all the denominations of money they can. Then show them a nickel and ask how many pennies could be exchanged for one nickel. On the chalk board write:

1 nickel = 5 pennies = 5¢

Take five pennies in one hand and five pennies in the other, and then put them together. Using repeated addition, develop the following table:

1 nickel = 5 pennies = 5¢

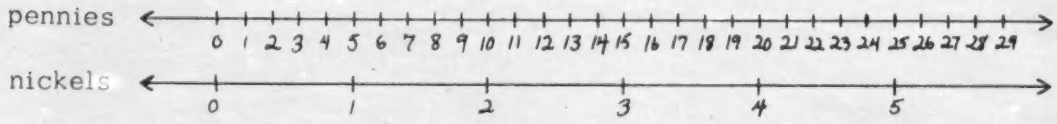
2 nickels = (5¢ + 5¢) = 10¢ = 10 pennies

3 nickels = (5¢ + 5¢ + 5¢) = 15¢ = 15 pennies

4 nickels = (5¢ + 5¢ + 5¢ + 5¢) = 20¢ = 20 pennies

5 nickels = (5¢ + 5¢ + 5¢ + 5¢ + 5¢) = 25¢ = 25 pennies

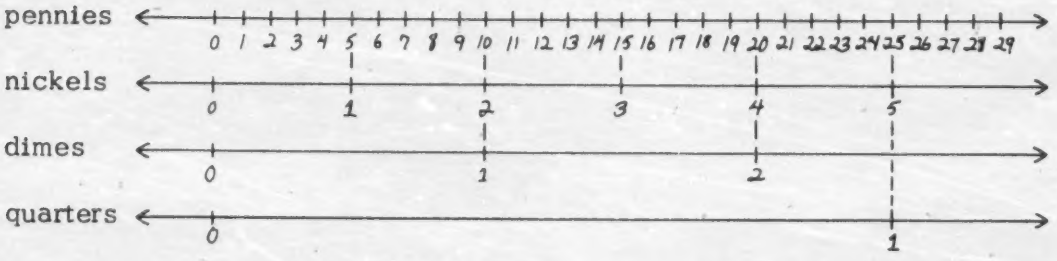
Near the top of the chalk board draw a number line labeled to 25. At the left, label the line "pennies." Below this line draw another line labeled "nickels." When you mark off the units on this line, be sure that your 0 is directly under the 0 on the pennies line and that your 1 is directly under the 5 on the first line, and so on.



Show the children a dime and ask how many pennies it is worth. Develop a table for dimes in the same way you did for nickels, continuing as long as you consider necessary.

Draw a number line below the first two, for dimes. Be sure your 1-point is directly below the 10-point on the penny line and the 2-point on the nickel line.

Show the children a quarter and ask how many pennies it is worth. Develop a table and then a number line as you did for the other coins. Your number lines should now look like this:



Worksheet 59
Unit 16 Name_____

Use your addition slide rule or the number line to help you solve these problems.

17 pennies + 9 pennies = 26 pennies = 26 ¢

24 pennies + 4 pennies = 28 pennies = 28 ¢

63 pennies + 8 pennies = 71 pennies = 71 ¢

5¢ + 13¢ = 18 ¢

21¢ + 9¢ = 30 ¢

93¢ + 4¢ = 97 ¢

16¢ + 4¢ = 20 ¢

32¢ + 10¢ = 42 ¢

To illustrate the interrelations among the coins, draw dotted vertical lines as shown. Point to 5 on the first line and ask what coins could be used to represent the number of cents. (5 pennies or 1 nickel.) Repeat with 10, 15, etc. Continue until you think the children are ready for Worksheets 59, 60, and 61; and then have the worksheets completed. Do not have the children do the starred problems on Worksheets 60 and 61 unless you have presented the necessary class work.

After completing the worksheets, if the children wish to play with the printed coins in their Student Manuals, have them paste the sheet on a piece of light cardboard or construction paper and then cut out the coins.

Worksheet 60
Unit 16 Name_____

10¢ = 1 dimes = 2 nickels = 10 pennies

20¢ = 2 dimes = 4 nickels = 20 pennies

60¢ = 6 dimes = 12 nickels = 60 pennies

* 50¢ = 1 half dollars = 2 quarters = 50 pennies

*100¢ = 2 half dollars = 4 quarters = 100 pennies

15¢ = 3 nickels = 15 pennies

35¢ = 7 nickels = 35 pennies

45¢ = 9 nickels = 45 pennies

Worksheet 61
Unit 16 Name_____

1 dime + 1 quarter = 10 ¢ + 25 ¢ = 35 ¢

2 nickels + 2 dimes = 10 ¢ + 20 ¢ = 30 ¢

4 pennies + 1 quarter = 4 ¢ + 25 ¢ = 29 ¢

3 pennies + 6 pennies = 3 ¢ + 6 ¢ = 9 ¢

*1 half dollar + 1 dime = 50 ¢ + 10 ¢ = 60 ¢

*2 quarters + 3 nickels = 50 ¢ + 15 ¢ = 65 ¢

TOR SAYS GOODBYE

At the end of the lesson about coins, Miss Johnson noticed that Tor wanted to say something. He was waving his hand around in the air for her attention.

"What is it, Tor?" she asked. "Do you want the children to explain more about our money?"

"No," Tor said, "What I want to say is that I have to go back to Titan now -- at least for a while -- and I have to say goodbye. Thank you all for teaching me so much."

Tommy thought he saw tears in Tor's eyes. "Don't go, Tor," he said, "we can still teach you about many, many more things."

Then the children all gathered around Tor, begging him not to leave. Sally said, "We like you so much, Tor!" and Joe said, "It's so much fun learning with you," and John said, "Why do you have to go, anyway?"

Tor said, "I have to go now, while I still have enough food and other supplies left for my journey. I have never learned to eat your food, you know, even though I have tried."

"Tor's right," Mary said, "he can't even eat cookies. They don't agree with him at all."

The children groaned. It seemed very sad that their friend could not eat cookies.

Miss Johnson said, "We want to thank you, Tor, for coming to visit us. It has been a great pleasure to have you here. We appreciate it so much that we certainly will not try to make you stay when we understand why you have to go home."

Tommy said, "Tor, how soon do you have to leave?"

"It will take me a little while to check my space ship," Tor said, "but then I must take off."

"Well, check your ship and then come back -- at least for a minute," Tommy said.

While Tor was checking his ship, inside and out, the children and Miss Johnson were feverishly planning how they should say goodbye to their little friend.

"I'm going to draw a picture for him, "Sally said, and she got busy with her crayons.

"I'm going to give Tor my shiny penny," Tommy said.

"Let's put whatever we can give Tor on the display table, Joe said, "and let him take his pick."

"And when he leaves we will sing 'Happy Journey' to him," Eddie said.

"I never heard of that song," Jack said. "How does it go?"

"It's just like 'Happy Birthday,' only you sing, 'Happy Journey, dear Tor' instead."

"Good idea!" Jack said, and the other children agreed.

When Tor returned, the children were all waiting for him. Tommy showed Tor the presents on the display table. "You can have whatever you want to take back to Titan with you," Tommy explained. "Some of the children drew pictures for you; others gave small things they thought your friends on Titan might like to see. We know you don't have space for anything very large."

Tor's eyes shone as he looked at the gifts. "Pictures," he exclaimed happily, "and pencils! Paper clips! Crayons! Chalk! A shiny penny! Oh, what a wonderful selection! What fun I will have showing these things back home! My friends on Titan have never seen anything like them!"

"You will need this box to put your presents in," Sally said, as she gave him an empty carton.

"My, you think of everything!" Tor said. Then he began to pack the box. The children clustered around, watching. Tor admired each picture before he placed it in the bottom of the box. Next, he carefully placed all the small gifts on top of the pictures. Then, much to everyone's surprise, Tor put in an addition slide rule, an abacus, and even a beam balance that just happened to be be left on the display table.

Tor said, "Thank you" again and again and the children said, "Hurry back, Tor!" "Don't forget us, Tor!" and "Say hello to your friends on Titan!"

When everything was packed, Tor picked up the box and went to the window. The children followed, singing the Happy Journey song. They watched and sang as Tor sprayed himself out to his space ship. They waited at the window while he put the box inside the ship. Then he came out once more, and stood on the ladder to wave goodbye.

Everyone waved back, and shouted "Goodbye" and "Good luck." And everyone watched as the little space ship took off. They watched and watched until the ship was only a little speck in the sky. Finally, when they could not even see the speck anymore, Mary said, "Miss Johnson, did you expect Tor to take the

abacus, the slide rule and the beam balance?"

"No, they just happened to be left there on the table." Miss Johnson blushed. The children smiled, enjoying the joke.

Then Sammy said, "I'm glad you left them there. I think it was lucky, because anyone could see that Tor really wanted them."

"I'm glad, too," Miss Johnson said. "I think Tor will make good use of them."

Tommy said, "Miss Johnson, do you think Tor will come back back to visit us again?"

"I don't know, Tommy. He seems to have a very fast and efficient space ship, but he never could explain how it was made or how long it took him to get here."

"I hope he does come back sometime," Sally said. "It was fun explaining things to him."

Tommy said, "Well, who knows? Maybe some day we will have such a good space ship that we can go and visit him."

Some of the children laughed, but Sammy said, "That just might be possible, you know." His remark seemed to brighten every face in the room.